

University of Oslo
Department of Philosophy

The Semantic Anti-Realist Challenge

**An exposition of the dispute between
Semantic Realism and Anti-Realism**

Master Thesis in Philosophy

Supervisor: Carsten Hansen
Student: Aleksandar Katanovic
Oslo, spring 2002

Acknowledgments

During my studies of philosophy, there are three people whose lectures and works have greatly influenced and shaped my philosophical views.

Dr. Carsten Martin Hansen my supervisor, for his guidance, stimulation and support. His seminars had aroused my interest for Michael Dummett's works.

Dr. Camilla Serck-Hansen and Dr. Jens Saugstad, for their lectures on Kant's epistemology. Although their lectures were held some years ago, I still hold them very dear in my memory.

I wish also to thank Andreas Ribe, my friend and fellow student, for many discussions encouraging me to work with the thesis.

Lastly to my wife Hedvig Buene, for her patience and loyal support, without which I would not have accomplished this work.

Oslo, April 30, 2002

Aleksandar Katanovic

CONTENT

INTRODUCTION.....	1
A. Background of the Semantic Anti-Realist Challenge	3
1. Introduction.....	3
2. The Semantic Anti-Realist Challenge.....	4
3. Verificationism and Intuitionism	9
4. Resume.....	16
B. Dummett's Three Arguments.....	17
1. The Communication Argument	17
2. The Manifestation Argument.....	22
3. The Acquisition Argument	25
4. The relation between the three ideas of arguments.....	27
C. Evaluation of Arguments and Their Role in the Formulation of ARC.....	29
1. General Observations.....	29
2. The Manifestation Argument (Arg2).....	32
3. The Communication Argument (Arg1)	33
4. The Acquisition Argument (Arg3a).....	35
5. ARC seen in the light of the three arguments.....	39
D. The Platonist Reply to ARC.....	42
1. Introduction.....	42
2. Platonist and Intuitionist Correctness Condition Constraints	44
3. The neutral interpretation of CC and difficulties in the R/AR dispute	46
4. Some conceptual distinctions.....	48
5. The first type of dispute	49
6. The second type of dispute	52
7. Evaluation of George's reply	56
E. The Holistic Realist Reply to ARC	65
1. The general meaning theoretic thrust of ARC	65
2. Truth in Loar's Holistic Theory.....	66
3. Realism and Truth.....	70
4. Realism and Meaning	73
5. The Realist Reply.....	75
a) Rejection of the truth-conditional approach	75
b) The first stage of argumentation.....	77
c) The second stage of argumentation	81
6. Evaluation of Loar's Reply	86
CONCLUSION.....	93
BIBLIOGRAPHY.....	95

INTRODUCTION

The objective of the thesis is to give one account of Dummett's philosophical approach in the justification for intuitionistic logic. Dummett's argumentation is worth being considered not merely in its attempt to repudiate the Platonist understanding of mathematics in favor for Intuitionism, but it also provides one perspective of the nature of disagreement between Platonism and Intuitionism in the philosophy of mathematics.

In the course of the exposition, we will learn that Dummett's justification for intuitionistic logic takes the form of a challenge known as the Semantic Anti-Realist Challenge (ARC). The central theme of the thesis is an enquiry of the argumentative strength of the Semantic Anti-Realist Challenge.

The first chapter serves as an introduction giving a general background of ARC, and the content of ARC. Here we are concerned with the underlying principles of ARC.

The second chapter is a hermeneutic exposition, in an analytic tradition, of the arguments supporting the principles of ARC. In the exposition, we try to find interpretations which would render the arguments as strong as possible.

The third chapter is an evaluation of the arguments and their role in the formulation of ARC is being examined.

In the fourth chapter, we are examining the Platonist reply to the Challenge of ARC. Our examination is based on Alexander George's paper "How not to refute realism." His paper is worth to take into account, in that it considers a Platonist reply, which accepts the underlying principles of ARC. The dispute between a Platonist and an Intuitionist has the character of a dispute between realism versus anti-realism (R/AR dispute) concerning mathematical objects.

In the fifth chapter, we will deal with one Holistic Realist reply to ARC. It differs from the first reply in that it is not concerned with the particular R/AR dispute between Platonists and Intuitionists in the philosophy of mathematics, but rather with the general meaning theoretic considerations. Although we are most concerned with the R/AR dispute in the philosophy of mathematics, we cannot ignore the Holistic Realist reply,

since the force of the Semantic Anti-Realist Challenge is heavily based upon general meaning theoretic considerations and has, thus, *prima facie* a global anti-realist character not limited within the R/AR disputes in the philosophy of mathematics. The holistic realist reply in question is Brian Loar's reply, and our examination of his reply will be based upon his paper "Truth Beyond All Verification."

In the exposition, we are concerned with two central questions:

- The validity of the underlying principles of ARC
- The force of ARC

The first question is concerned about the cogency of the arguments for the underlying principles of ARC. The second question is concerned whether the Challenge of ARC can be met by a realist, given that the principles of ARC are accepted.

A. Background of the Semantic Anti-Realist Challenge

1. Introduction

The object of the thesis is to give one account of Dummett's philosophical approach in the justification for intuitionistic logic. In the course of the exposition, we will consider Michael Dummett's paper "The Philosophical Basis of Intuitionistic Logic," where one argument is provided for rejecting classical logic¹ in favor of intuitionistic logic. Dummett's argumentation is worth being considered not merely in its attempt to repudiate the Platonist understanding of mathematics in favor for Intuitionism, but it also provides one perspective of the nature of disagreement between Platonism² and Intuitionism in the philosophy of mathematics.

Dummett's approach in the justification of intuitionistic logic is based on one verificationist meaning theory. His argumentation is, in other words, based on some considerations on language, in contrast to the traditional intuitionist arguments. Traditional intuitionist arguments take rather a metaphysical approach, where the metaphysical anti-realist³ assumption on the nature of mathematical objects, namely that mathematical objects do not have an objective reality existing independently of us, serve as a premise for repudiating the Platonist perspective of mathematics. Dummett questions the merit of such metaphysical anti-realist arguments for intuitionism, and favors his semantic anti-realist approach.

We will examine Dummett's semantic approach for intuitionism. This examination can provide a basis for investigating the problem on the relation between the

¹ "Classical logic" will be understood as a logic that accepts all classical logical laws. Logical laws in question are (a) the law of excluded middle, with its corresponding semantic principle, the principle of bivalence, (b) the law of non-contradiction, with its corresponding semantic principle, and (c) the law of double negation with its corresponding semantic principle. In the preface to *The Truth and Other Enigmas*, Dummett explains his terminology of the logical laws, which is adopted in my thesis. Dummett is consistent in his terminological usage from his early to his late works. Thus, we can see that in his last work, *The Logical Basis of Metaphysics*, Dummett distinguishes classical from non-classical logical systems by observing whether logical laws and/or their classical semantic interpretations are accepted.

² "Platonism" should be understood in this paper only in the context of philosophy of mathematics, as a position that consider mathematical objects as abstract objects existing independently of mind.

³ Metaphysically anti-realist assumption with respect to mathematical objects only.

metaphysical approach and the semantic approach: does a metaphysical perspective on the nature of mathematical objects depend on the meaning theoretic considerations on mathematical statements? The question might be investigated by examining what kind of metaphysical consequences follow from Dummett's arguments for intuitionism, if we take that his arguments are cogent.⁴ In this thesis, however, our task will only be concerned with the cogency of Dummett's argument for intuitionistic logic.

2. The Semantic Anti-Realist Challenge

What I take as the heart of Dummett's argument is the following line of reasoning (ARC):

[ARC]

[Platonist] conception violates the principle that use exhaustively determines meaning (...) For, if the knowledge that constitutes a grasp of the meaning of a sentence has to be capable of being manifested in actual linguistic practise, it is quite obscure in what knowledge of the condition under which a sentence is true can consists, when that condition is not one which is always capable of being recognized as obtaining.⁵

The heart of Dummett's argument is a challenge for a Platonist to show that the Platonist conception does not violate "the principle that use exhaustively determines meaning." How does Dummett expect from a Platonist to meet the challenge? It is expected that a Platonist explains what the understanding of the truth-conditions of mathematical statements consists in, when generally those truth-conditions of mathematical statements are not always capable of being recognized as obtaining. This challenge is a semantic anti-realist, since it challenges the realist conception of truth. The semantic realist conception of truth is a view that takes the truth-value of a statement as objectively existing, independently of the verification of the statement. The semantic anti-realist conception of truth, on the other hand, is a view of truth that truth is verifiability; i.e. a

⁴ It is this question which is of uttermost concern for Dummett. Dummett asks what is the relation between these two approaches. For instance, could we hold consistently a metaphysical realist position with respect to mathematical objects, but at the same time reject classical logic in favour for intuitionistic logic?

⁵ Dummett, "The Philosophical Basis of Intuitionistic Logic" in: *Truth and Other Enigmas*, p. 224

statement is true if and only if it is verifiable, that is, if and only if there is evidence warranting its assertion.⁶

This verifiability of a statement is not understood in a narrow empiricist sense, as understood by logical positivists. As Dummett says, “the verification would not ordinarily consist in the bare occurrence of some sequence of sense-experience, as on the positivist conception of the verification of a statement.”⁷ Generally, verification of a statement is understood as establishing it as true by a process of reasoning (argument), where the reasoning will not usually be purely deductive in character. The premises of the argument will be based on observation. “In the mathematical case, that which establishes a statement as true is the production of a deductive argument terminating in that statement as conclusion.”⁸

What is important in Dummett’s idea of verifiability is the idea of being capable of recognizing an argument that conclusively establishes a sentence as true, when presented with the argument. As we shall see, the question of the interpretation and application of this idea of verifiability in the theory of meaning is the main problem in the dispute between Platonists and Intuitionists.

The dispute between a Platonist and an Intuitionist is about the question of what the *understanding* of a mathematical statement consists in. According to a Platonist, understanding of a mathematical sentence consists in knowing its truth-condition, even though the condition is one which in general cannot be recognized as obtaining when it does obtain. However, an Intuitionist à la Dummett would challenge the intelligibility of the Platonist view, as seen in ARC. What does the knowledge of truth-condition consists

⁶ A weaker version of semantic anti-realism does not maintain that verifiability of a statement is sufficient for the truth of the statement, but that a truth of a statement entails that the statement is verifiable. However, we will first focus our attention on the stronger version when discussing the relation between truth and verifiability of mathematical statements, where verifiability of mathematical statements consists in their provability. In such context, Dummett’s anti-realist takes the stronger version of semantic anti-realism. The weaker version will be discussed when considering generally the question of the relation between truth and verification applied to other assertoric statements, e.g. empirical statements, statements about someone’s character traits, statements about past state of affairs, etc.

⁷ Dummett, op cit, p. 227

⁸ Dummett, op cit, p. 227

in when “that condition is not one which is always capable of being recognized as obtaining”? The thrust of ARC is the idea that a use theory of meaning must take a form of a verificationist theory of *meaning* (verificationism)⁹, and what Dummett tries to establish is that verificationism implies the verificationist conception of *truth*. There are two issues concerning the force of ARC. First, whether a use theory implies a verificationist theory of meaning. Second, whether verificationism implies the verificationist conception of truth.

For some philosophers, as Skorupski, it is not apparent that verificationism implies the verificationist view of truth.¹⁰ As Skorupski defines it, “verificationism is the view that understanding a statement consists in grasping what information states would verify it.”¹¹ This could be quite compatible with a Platonist view of the meaning of mathematical sentences, if “grasping what information states that *would* verify” a mathematical sentence can be interpreted as having understanding of (=“grasping ...”) truth-condition for the mathematical sentence (=“... what information states ...”) should be obtained in order for them to be true (=“... would verify them”), but that the truth-condition is not being always capable of being recognized as obtaining. A Platonist maintains that there is a distinction *between* grasping what information state would verify a mathematical sentence *and* being in that information state. Consider the Goldbach’s conjecture, which asserts of every natural number that, if it is even and greater than 2, then it is the sum of two prime numbers. The information state that would verify the Goldbach’s conjecture is the observation for each natural number that it has the property G.

[G]

If it is even and greater than 2, then it is the sum of two prime numbers.

⁹ The term ‘verificationism’ will be restricted only to the verificationist view of *meaning*, and not to the verificationist view of *truth*. Because the verifiability of a statement is not understood in a logical positivist sense, Dummett’s verificationism should not be confused with verificationism of logical positivists.

¹⁰ See John Skorupski, “Meaning, use, verification”, in: *A Companion to the Philosophy of Language*, pp. 40-42.

¹¹ Skorupski, op cit, p. 41.

A Platonist maintains that we understand the meaning of Goldbach's conjecture, because we understand what we must observe in order to verify Goldbach's conjecture. That we human beings cannot observe for each natural number that it has the property G, if it has, does not worry a semantic realist, since this is precisely the realist point, namely that there is a distinction between knowing what kind of observation would verify a mathematical sentence and being in a position to observe whether the mathematical sentence is true. Furthermore, there might not be any finitely expressible explanation why each natural number has the property G, if it in fact has, because the truth-conditions of the realist are such that infinite coincidences are intelligible possibilities. What comes here in play is the notion of infinity as conceived as a complete totality, fundamental to classical mathematics. Infinite sets are understood as complete total domains, and if the notion of iterative set that depends on the metaphor of the process is taken as unproblematic, a realist would affirm that infinite processes are completable, e.g. that it is possible to collect together all members of an infinite set.

A Platonist notion of infinity is in deep conflict with the intuitionistic understanding of infinity. An intuitionist notion of an infinite set is a process of construction of members of the set that never takes end, and as such it is never completable. As such, an infinite set is not one totality of elements that already exists, but rather it consists of elements that are brought in being through a mathematical activity that cannot come to an end. The existence of all but finitely many of them must always remain merely potential. Therefore, the intuitionistic notion of infinity is called the potential infinity. As a consequence, what an intuitionist would object to a realist in the case of Goldbach's conjecture is not in the particular operations on checking procedures that a realist would perform, but in "the realist's conception of the domain upon which this checking procedure operates."¹² What an intuitionist objects to is the question of the intelligibility of the Platonist notion of infinity.

ARC is a general attempt to establish the link between the verificationist view of meaning and the verificationist view of truth, by trying to show that a Platonist

¹² See Alexander George, "How not to refute realism", in: *The Journal of Philosophy*, vol. XC, No. 2, Feb. 1993. p. 61.

interpretation of “grasping what information states that would verify” a mathematical statement is an unintelligible one.

We will be concerned with three problems for ARC, and correspondingly to them three tasks. Firstly, the analysis of Dummett’s argumentation for verificationism, argumentation preceding ARC.¹³ Secondly, the analysis of Dummett’s verificationist view of meaning conveyed in his argumentation and, thirdly, whether his view, taken as a premise, can establish the verificationist conception of truth through ARC. What is important about the second task is to investigate the nature of the argumentative force of Dummett’s verificationist premise: (a) whether it is a neutral one, such that a semantic realist can accept it, not being a question begging premise, or (b) whether the premise is a compelling one so that a semantic realist is obliged to accept it, no matter if it contradicts with a realist framework, or (c) whether it has none of the argumentative features described in (a) and (b). The next section will first deal with the preliminaries of the third task, before taking the first and the second ones into account. My motivation for beginning with the third question is that it will give a preliminary idea of how ARC establishes the link between verificationism and the verificationist view of truth when dealing with mathematics. By doing so, I hope that we might understand the motivations behind the argumentation preceding ARC.

Let us take again a look on ARC. In the stated challenge there are two premises:

[UP]

The use of a sentence exhaustively determines its meaning

[MP]

The knowledge that constitutes a grasp of the meaning of a sentence has to be capable of being manifested in actual linguistic practise

In the structure of the challenge, UP is being taken as a principle that implies MP. In the first part of Dummett’s paper, preceding ARC, there are three arguments whose object is to establish that “the knowledge that constitutes a grasp of the meaning of a sentence has to be capable of being manifested in actual linguistic practise,” by showing

¹³ Two things will be argued for: (1) arguments preceding ARC are arguments that try to establish premises figuring in ARC, and (2) these premises imply a compositional verificationist theory of meaning.

that UP is a valid principle and that the validity of UP implies MP. The first argument will be called the Communication Argument. The second argument will be called the Manifestation Argument. The third will be called the Acquisition Argument. The examination of these three arguments is the object of the first task. Before proceeding with the first task, we will consider two questions concerning the relation between ARC, verificationism and intuitionism.

3. Verificationism and Intuitionism

From the foregoing it was said that ARC tries to show that verificationism implies a verificationist view of truth. How does ARC show that verificationism implies a verificationist view of truth, if we accept both UP and MP? More importantly, how does verificationism enter into the picture when considering ARC? What follows is a preliminary account of intuitionistic perspective regarding these two above questions. Afterwards we will take a more critical look at it, and see whether it can be improved.

The knowledge that constitutes a grasp of the meaning of a mathematical sentence is manifested in an observable practise. Which practise? A practise that shows how we learn mathematics: in learning mathematics we acquire mastery of how to use mathematical statements. Therefore, “it is in the mastery of that practise that our grasp of the meanings of the statements must consist.”¹⁴ And what do we actually learn to do when we learn some part of mathematics? Dummett answers us: “What we actually learn to do, when we learn some part of the language of mathematics, is to recognize, for each statement, what counts as establishing that statement as true or false.”¹⁵ Therefore, the meaning of a mathematical statement consists in recognition, for the statement, what counts as establishing that statement as true or false. In other words, the notion of *proof* is the central notion of the theory of meaning of mathematical statements:

¹⁴ Dummett, op cit, p. 225

¹⁵ Ibid

[RP]

A grasp of the meaning of a statement consists in a capacity to recognize a proof of it when one is presented to us.¹⁶

A mathematical proof is a verification of a mathematical statement.

If the above verificationist reasoning is sound, how does it imply a verificationist view of truth? What was argued for was that the *meaning* of a mathematical statement consists in the recognition of its proof (verification) when one is presented to us. Implicit in Dummett's verificationism is a view of language that accepts the principle of compositionality, which in Dummett's compositional meaning theory takes the following idea:

[CP]

A grasp of the meaning of any expression smaller than a sentence consists in knowledge of the way in which its presence in a sentence contributes to determining what is to count as a proof of that sentence.¹⁷

The compositional verificationist view of meaning seems to imply the verificationist view of truth by considering the question of the meaning of logical constants. In other compositional meaning theories, the principle of compositionality would be formulated differently, as Dummett illustrates it with logical constants.

The principle of compositionality is most easily illustrated by the logical constants. On a compositional meaning-theory, to know the meaning of 'or', for example, is to be able to derive, from the meanings of any sentence **A** and **B**, the meaning [**A or B**], where the meaning of a sentence consists in what counts as verifying it [verificationist theory], or in the consequence of accepting it as true [pragmatist theory], or in the condition for it to be true [truth-conditional theory]. To understand [**A or B**], therefore, you must (i) observe the composition of the sentence, (ii) know what 'or' means, and (iii) what **A** and **B** mean, whereas the third component of an understanding of [**C or D**] will be different, namely knowing what **C** and **D** mean.¹⁸

In a verificationist compositional meaning theory, the meaning of a logical constant '*' can be interpreted as LCV, which embodies the idea of CP.

¹⁶ Ibid

¹⁷ Ibid

¹⁸ Dummett, *The Logical Basis of Metaphysics*, p. 222

[LCV]

To know the meaning of a logical constant ‘ $*$ ’ it has to be able to derive, from the meanings of any sentence **A** and **B**, the meaning **[A * B]**, where the meaning of a sentence consists in what counts as verifying it.

LCV is nothing else than an intuitionistic interpretation of logical constants. The intuitionistic explanation of the meaning of logical constants can be alternatively explained as following:

[ELC]

Meaning of each constant is given by specifying, for any sentence in which that constant is the main operator, what it is to count as a proof of that sentence, assuming that we already know what it is to count as a proof of any of the constituents¹⁹

An important thing to note here is that the intuitionistic explanation of logical constants is faithful to the following principle:

[PIM]

For any construction that is presented to us, we shall always be able to recognize effectively whether or not it is a proof of any given statement²⁰

The principle PIM is regarded as the most important principle of intuitionistic mathematics, and Dummett writes that the “assumption that we can effectively recognize a proof of a given statement of some mathematical theory, say elementary number theory, lies at the basis of all intuitionistic mathematics.”²¹ Why is an intuitionist insistent on PIM? How can we be able to recognize effectively whether any construction is a proof of any given statement? The answer is that a grasp of the meaning of a statement consists in a capacity to recognize a proof of it when one is presented to us. In other words, PIM is justified by the principle RP.

Given ELC, we arrive at the verificationist view of truth, by considering the meaning of the logical constant ‘or’, where it occurs as the main operator, and the meaning of the schema **[A \vee \neg A]**. Meaning of **[A \vee B]** is that we have a proof of **[A \vee B]**

¹⁹ See Dummett, *The Elements of Intuitionism*, p. 12

²⁰ Ibid

²¹ Dummett, op cit, p. 264

by either having a proof of **A** or by having a proof of **B**. Proof is understood as a *finite* construction, e.g. a computation or finite set of computations, involving only the logical constant in question. If we substitute **B** with the negation of **A**, the statement $[A \vee \neg A]$ would mean that we have a proof of $[A \vee \neg A]$ by either having a proof of **A** or by having a proof of its negation. Clearly, we cannot be entitled to say that we have a proof of $[A \vee \neg A]$, since we in general are not in the possession of either the proof of **A** or of its negation. Thus, the principle of exclude middle and its corresponding semantic principle, the principle of bivalence, are not intuitionistically valid principles. In other words, we are not entitled to say that for any mathematical sentence, it has a determinate truth-value, unless we have a proof or a refutation of it. As a consequence, a statement is true if and only if it is verifiable.

There are the following problems with establishing a verificationist view of truth from verificationism.

First, ARC presupposes a compositional meaning theory, and it does not take into account non-compositional meaning theories. A holistic meaning theory is an example of a non-compositional theory: a sentence's meaning is not explained in terms of its composition but rather in terms of its role in the language. However, Dummett's philosophical justification for intuitionistic logic is based on some general meaning theoretic considerations. Given the general thrust of Dummett's argumentation, the argumentation preceding ARC, ARC cannot be restricted only to the context of the dispute between Intuitionists and Platonists concerning the relation between truth and verifiability of mathematical statements. Both an Intuitionist and a Platonist hold that the application of the predicate 'true' to a mathematical sentence must be explained in the terms of composition of the mathematical sentence. However, a non-compositional meaning theoretic approach and other classes of statements, as empirical ones or statements about the state of affairs in the past, should also be considered.

A second problem is whether the interpretation of the idea of verifiability employed by an intuitionist is the one that a Platonist would accept? Might the intuitionist notion of proof incorporate concepts that are central in the dispute, as for instance in the possibility/impossibility of proofs involving quantification over infinite domains, as seen in the case of Goldbach's conjecture? If so, then a Platonist might accept the correctness

condition constraint upon the understanding of mathematical sentences. The correctness condition constraint on the understanding of mathematical sentences is a constraint restricting the choice of correctness conditions for sentences in a mathematical discourse, and can be formulated as CC.

[CC]

Understanding of a mathematical sentence consists in associating correctness conditions for the sentence in such a way that the association is paired with a fully exercisable capacity to recognize that these correctness conditions obtain when they do.²²

Alexander George argues that accepting the correctness condition constraint would not imply a verificationist theory of truth under some interpretations of the constraint, interpretations in harmony with the Platonist view of mathematics. We will consider Alexander George's article "How not to refute realism," which examines the question whether a Platonist can accept CC.

Dummett's contention is that the acceptance of the correctness condition constraint CC is incompatible with a Platonist view of mathematics. Dummett would say that the only meaning theory available for the Platonist view is the truth-conditional theory, where truth-conditions can transcend the verifiability conditions. However, the truth-conditional theory, Dummett argues, would be in conflict with the principle UP. The thrust of ARC is that the acceptance of the principle UP will ultimately justify the acceptance of RP. What is the logical relation between CC and RP? If 'a fully exercisable capacity to recognize that the correctness conditions for a mathematical sentence obtain when they do' is interpreted as FEC, then CC is valid if and only if RP is valid; $CC \Leftrightarrow RP$ under the interpretation FEC:

[FEC]

A fully exercisable capacity to recognize that the correctness conditions for a mathematical sentence obtain when they do is a capacity to recognize a proof of it when one is presented to us.

[CC \Rightarrow RP]: (1) A fully exercisable capacity to recognize that the correctness conditions for a mathematical sentence obtain when they do is a capacity to recognize a

²² Alexander George, op. cit., p. 54

proof of it when one is presented to us. (2) Understanding a mathematical sentence consists in associating correctness conditions for the sentence. (3) The association of correctness conditions for a sentence is paired with a fully exercisable capacity of recognizing that these conditions obtain when they do. (4) The association of correctness conditions for a sentence is paired with a capacity to recognize a proof of it when one is presented to us. (5) Understanding a mathematical sentence consists in a capacity to recognize a proof of the mathematical sentence. (6) Understanding a mathematical sentence is the same as to grasp the meaning of a sentence. (7) A grasp of the meaning of a statement consists in a capacity to recognize a proof of it when one is presented to us.

[RP \Rightarrow CC]: (1) A capacity to recognize a proof of a mathematical sentence when presented with one is a fully exercisable capacity to recognize that the correctness conditions for a mathematical sentence obtain when they do. (2) A grasp of the meaning of a statement consists in a capacity to recognize a proof of it when one is presented with one. (3) A grasp of the meaning of the sentence consists in a capacity of recognizing that correctness conditions of the sentence obtain when they do. (4) A grasp of the meaning of a mathematical sentence is the same as understanding the sentence. (5) Understanding a mathematical sentence consists in a fully exercisable capacity of recognizing that correctness conditions for the sentence obtain when they do. (6) A fully exercisable capacity of recognizing that correctness conditions for a mathematical sentence obtain when they do implies that these correctness conditions were associated with the sentence in order to understand the meaning of the sentence. (7) Understanding of a mathematical sentence consists in associating correctness conditions for the sentence in such a way that the association is paired with a fully exercisable capacity to recognize that these correctness conditions obtain when they do.

Explanation of the steps involved in the argument [CC \Rightarrow RP]:

(1) is a premise based on the interpretation FEC. (2) follows from CC. (3) follows from CC. (4) follows from (1) and (3). (5) follows from (2) and (4). (6) is a premise unproblematic in the dispute. (7) follows from (5) and (6).

Explanation of the steps involved in the argument [RP \Rightarrow CC]:

(1) is a premise based on the interpretation FEC. (2) is RP. (3) follows from (1) and (2). (4) is a premise unproblematic in the dispute. (5) follows from (3) and (4). (6) follows from (5) and a reflection on what is involved when possessing a fully exercisable capacity of recognizing that correctness conditions for a sentence obtain when they do. (7) follows from (5) and (6).

Both above arguments, necessary for establishing $[CC \Leftrightarrow RP]$, hinge on their first premise. If ‘a fully exercisable capacity to recognize that these correctness conditions obtain when they do’, as figuring in the formulation of CC, is interpreted as possessing an effective procedure enabling to recognize that these correctness conditions obtain when they do, then both above arguments would not be valid. A capacity to recognize a proof of a mathematical sentence when presented with one does not imply a capacity to produce such a proof. Having an effective procedure enabling to recognize that correctness conditions for a mathematical sentence obtain when they do is a procedure to produce a proof of the sentence. Such interpretation, which invalidates equivalency between CC and RP, should not be attributed to the intuitionistic view on the relevant cognitive capacity. An intuitionist acknowledges the existence of effectively undecidable mathematical statements; statements which have neither been proved nor refuted, and for which we lack effective means that would yield either a proof or refutation of these statements.²³

²³ I am following Dummett’s usage of the term ‘effectively undecidable statement’. However, in a standard mathematical practise, “decidability is a property in the first instance of *a set of statements*. Such a set M is said to be decidable just in case M’s characteristic function is recursive.” See more on this terminological difference in Alexander George, op. cit., footnote 2.

4. Resume

We have considered the intuitionistic perspective on the relation between verificationism and the verificationist view on truth. The meaning of the mathematical statements is subject to the correctness condition constraint CC. Dummett's contention is that the acceptance of the correctness condition constraint is incompatible with a Platonist view of mathematics. Dummett would say that the only meaning theory available for the Platonist view is the truth-conditional theory, where truth-conditions can transcend the verifiability conditions. However, the truth-conditional theory, Dummett argues, would be in conflict with the principle UP. The thrust of ARC is that the acceptance of the principle UP will ultimately justify the acceptance of RP, which is equivalent to CC under the interpretation FEC.

In the next sections we will examine Dummett's arguments for both the principle UP and that UP implies MP. The motivation behind the examination is not merely in assessing their cogency but to understand the concepts involved in the argumentation, concepts which will elucidate the idea of ARC. Recall that UP and MP are two principles figuring as premises in ARC.

B. Dummett's Three Arguments

1. The Communication Argument

In the first part of Dummett's paper, preceding ARC, there are three arguments whose object is to establish UP and MP. We will consider in this section the first argument, which runs as follow, (I will number the sentences for ease of reference):

[Arg1]

(1) The meaning of a mathematical statement determines and is exhaustively determined by its *use*. (2) The meaning of such a statement cannot be, or contain as an ingredient, anything which is not manifest in the use made of it, lying solely in the mind of the individual who apprehends that meaning: (3) if two individuals agree completely about the use to be made of the statement, then they agree about its meaning. (4) The reason is that the meaning of a statement consists solely in its role as an instrument of communication between individuals, just as the powers of a chess-piece consists solely in its role in the game according to the rules. (5) An individual cannot communicate what he cannot be observed to communicate: (6) if one individual associated with a mathematical symbol or formula some mental content, where the association did not lie in the use he made of the symbol or formula, then he could not convey that content by means of the symbol or formula, (7) for his audience would be unaware of the association and would have no means of becoming aware of it.²⁴

The first line in defence for intuitionistic logic begins with asserting the principle UP. The key expression in the (1) is “exhaustively determined.” Why is the focus on the use of a sentence important for the overall argumentation in “The Philosophical Basis for Intuitionistic Logic”? This is because the use of a sentence is understood as a publicly observable practise. If the use of a sentence is something observable, then the proposition that the meaning of a sentence is exhaustively determined by something observable follows from (1). ‘The manifestation of meaning of a sentence’, which is the theme in (2), is understood as an observable phenomenon because the manifestation is connected to the use made of the sentence. This interpretation of ‘manifestation’ is further confirmed in the line (5) of the argument.

(1) is a conjunction of two propositions: (p1) The meaning of a sentence determines the use of the sentence, and (p2) the meaning of a sentence is exhaustively determined by the use of the sentence. The principle UP is logically equivalent with p2. (2) is particularly a consequence of (p2): if the meaning of a sentence is exhaustively determined by the use of the sentence then anything which is not made manifest by the

²⁴ Dummett, “The Philosophical Basis of Intuitionistic Logic,” p. 216

use of it neither constitute the meaning of the sentence nor is an ingredient of the sentence's meaning.

However, what does determine the correct use of a sentence? Answering the question by (p1) is not helpful. To say that the meaning of a sentence determines the correct use of the sentence does not explain it, if the meaning of the sentence was in the first place explained by the very usage of the sentence. If (1) represents an explanation of meaning, then (p1) and (p2) represent an explanatory circle: meaning determines the use, use determines the meaning. The circle is not necessarily a vicious one if some feature of either the meaning or of the use of a sentence can be exploited. (3) offers the answer to the problem: what determines the correct use of a sentence is the public agreement of how to use it.

The public agreement of how to use a sentence is a feature of the use of a sentence. This feature of the use of a sentence elucidates how the meaning of the sentence is observable: it is observable in the virtue of an agreement between speakers. If there is a complete agreement between speakers of how to use a sentence then the use of the sentence in accord to the agreement is made transparent between the speakers. What is the purpose of the agreement? The purpose is to have a communication. The explanation of what determines the correct usage of a sentence via the idea of communication is significant because it clarifies the relation between meaning and the use of a sentence; the explanatory circle (meaning \rightarrow use \rightarrow meaning) does not longer appear to be of a vicious character. The public agreement of the use of a sentence determines how to use the sentence, and this use in its turn determines the meaning of the sentence. Therefore, in understanding how the use of a sentence determines the meaning, we can also understand how "meaning of a sentence determines the use of it." The principle UP, therefore, represents a *general* heuristic idea of how the form of explanation of meaning of sentences should be given: the explanation of meaning of sentences should be given in the terms of the use of sentences in language.

The function of (3) in the argument is to illustrate the point of (1)-(2), and it also represents a transition to the main theme of the argument for UP, namely (4). (1) does not function as a premise of the argument but as the principle which is sought to be established. To repeat, Dummett's contention is that a truth-conditional theory of

meaning is not a use theory of meaning, because, he argues, the truth-conditional analysis of meaning would violate the principle UP. Therefore, Dummett is obliged to seek a justification for the principle UP. His first idea of defence for UP is the idea of communication. Why does the use of a sentence exhaustively determine its meaning? (4) is the answer: “The reason is that the meaning of a statement consists solely in its role as an instrument of communication between individuals.”

[COM]

The meaning of a statement consists solely in its role as an instrument of communication between individuals.

The problem with the answer is that (4) appears to be a question-begging argument if directed against some who does not accept UP. (4) is a stronger thesis about meaning than (1), because (1) does not specify which feature of the use is the relevant one in the determination of meaning. Consequently, (4) does not follow from (1). On the other hand, (1) follows from (4).

[COM \Rightarrow UP]: In saying that meaning of a statement *consists solely* in its role as an instrument of communication between speakers, there is no hidden aspect of meaning of statements in language which is *only* accessible to an individual speaker of the language, since such hidden aspect of meaning could not be communicated to other speakers, and as such could not serve the purpose of being communicated to other speakers. Communication involves an agreement of how to use statements in discourse. Therefore, the use of a statement, determined via public agreement serving the purpose of communication, exhaustively determines the statement’s meaning.

How shall we understand (5)-(7)? As I see it, there are two plausible readings of the argument:

Interpretation I of (5)-(7):

The purpose of (5)-(7) is to establish that (1) follows from (4). I do not see any reason for disputing (5). To communicate the meaning of a statement, whatever the ‘meaning of a statement’ is taken to be, involves that the meaning of a statement becomes observable for other speakers in the context of some discourse. Communication involves at least two speakers, and communication of the

meaning of a statement is successful if both speakers are able to observe the same association of content to the statement in their discourse. Therefore, a speaker cannot communicate what he cannot be observed to communicate.

How does the association of content of a statement become observable for speakers in a context of some discourse? More importantly, how can communication be at all publicly observable practise? (6)-(7) attempt to answer those above questions, namely that the association of some content to a (mathematical) statement lies only in the use of the statement in a (mathematical) discourse.

Given (5)-(7), (1) easily follows from (4). Since the meaning of a statement *consists solely* in its role as an instrument of communication between individuals, and the use of a statement is the only observable feature, which in the virtue of its observable character constitutes communication, the meaning of a statement is exhaustively determined by the use of it.

Interpretation II of (5)-(7):

In addition to establish that (1) follows from (4), (5) also serves as one reason for accepting (4), and (6)-(7) serve as a further explanation for (5).

Before considering the question of how can (5)-(7) be a justification for (4), let us first consider the motivation behind this second interpretation, by comparing it to the first one. The second interpretation is an extension of the first one; it agrees that the purpose of (5)-(7) is to establish that (1) follows from (4). However, the argument according to the first interpretation does not establish (4), crucial for establishing (1). Therefore, it should be considered whether (5)-(7) also serve to establish (4), interpreting the argument as strong as possible.

How can (5)-(7) be a justification for COM? COM is quite a strong thesis about the meaning of statements in that it contends that the meaning *consists solely* as an instrument of communication. It is an undeniable fact that language plays an important role as an instrument of communication, and consequently meaning of sentences in language serves to fulfil this role. But this fact is not sufficient to establish COM, because it must also be shown that communication is the *only* purpose of language. As Dummett

himself says in another paper: “A language has two functions, as an instrument of communication and as a vehicle of thought. One of the questions facing the philosophy of language is to determine which, if either, is the more fundamental.”²⁵ (5)-(7) refer to the undeniable fact of the social character of language as an instrument of communication. However, it seems that (6)-(7) do not rule out the existence of incommunicable concepts, which *might be* necessary in the concept formation of communicable concepts. Nevertheless, (5)-(7) have one important point: mathematical sentences are sentences used in a mathematical discourse, and by observing their use we observe their meaning. What (5)-(7) might have established, at least, is that meanings of some class of sentences consist solely in their role as an instrument of communication, and that the class of mathematical statements is such a class. Let us explore more of this issue, and see whether (5)-(7) have any merit for establishing COM.

The idea of (5)-(7) is that someone’s private association of content to an expression/statement²⁶, where the association does not lie in something publicly observable, i.e. the use of the expression, would not be the same association of content of the expression which his audience would understand. “His audience would be unaware of the [private] association and would have no means of becoming aware of it.”

The above idea is significant by considering the following questions about the private association of some content to an expression. Why should the private association of some content to an expression have the status of being meaning of *the* expression if the associated content of the expression is different from the content which his audience associates it with *the* expression? For instance, why should someone’s private association to the English expression “a cat”, which does not lie in the usage of referring to a member of the class of all cats, be *the* meaning of *the* expression? Would not the private association violate the rules of how to use the expression? The point is that not all associations of content determine the meaning of some expression, but rather that the *correct* association is the crucial one in the determination of the meaning.

²⁵ Dummett, “Truth and Meaning” in: *The Seas of Language*, p. 151

²⁶ In the text I will speak of expressions in language, but it applies also to statements.

What does determine the correct association of content to an expression? From the foregoing paragraph, we have considered what kind of problems emerge if private associations of content to expressions are permitted, *if* those expressions are belonging to a public language. What Dummett discusses is meaning of a particular class of expressions and statements belonging to a public language, namely the class of mathematical expressions and statements. Whether a speaker can privately associate content to an expression or statement belonging to a public language is not the issue. The issue is whether such private association can have any bearing in the determination of the meaning of a statement belonging to a public language. This will be examined when we deal with the evaluation of Dummett's arguments. To give a short resume of this section, the best interpretation of the Communication Argument is the Interpretation II, with a slight modification: Dummett is concerned with a public language. The question is whether COM is an acceptable principle for a public language.

2. The Manifestation Argument

The second argument runs as follows.

[Arg2]

(1) A model of meaning is a model of understanding, i.e. a representation of what it is that is known when an individual knows the meaning. (2) Now knowledge of the meaning of a particular symbol or expression is frequently verbalisable knowledge, that is, knowledge which consists in the ability to state the rules in accordance with which the expression or the symbol is used or the way in which it may be replaced by an equivalent expression or sequence of symbols. (3) But to suppose that, in general, a knowledge of meaning consisted in verbalisable knowledge would involve an infinite regress: (4) if a grasp of the meaning of an expression consisted, in general, in the ability to *state* its meaning, then it would be impossible for anyone to learn language who was not already equipped with a fairly extensive language. (5) Hence that knowledge which, in general, constitutes the understanding of the language of mathematics must be implicit knowledge. (6) Implicit knowledge cannot, however, meaningfully be ascribed to someone unless it is possible to say in what the manifestation of that knowledge consists: (7) there must be an observable difference between the behaviour or capacities of someone who is said to lack it. (8) Hence it follows, once more, that a grasp of the meaning of a mathematical statement must, in general, consist of a capacity to use that statement in a certain way, or to respond in a certain way to its use by others.²⁷

The first line of argument introduces another approach in the explanation of meaning. In explaining what the meaning of a statement is consisting of, we are explaining what the understanding of meaning consisting of. To understand sentences of

²⁷ Dummett, "The Philosophical Basis of Intuitionistic Logic," p. 217

some language we deal with their meaning. However, how shall we explain *this* understanding? This kind of approach is quite a challenging project. It involves an account of how an individual acquires an understanding of some language without presupposing that the individual has any knowledge of other languages. A theory endeavouring towards such goal is, like Dummett calls it, *a full-blooded theory of meaning*. “The model of meaning” in (1) is a model faithful to requirements of a full-blooded theory of meaning. Such interpretation is supported by observing two things what Dummett does in the course of his argument. First, it is denied that explicit knowledge has an explanatory value in the explanation of meaning. Second, the role of implicit knowledge is crucial in the explanation of meaning, with the consequence that it makes possible to give an account how an individual acquires an understanding of some language without presupposing that the individual has any knowledge of other languages.

If Dummett’s approach would be fruitful in the explanation of what the meaning of sentences of a certain language consists of, the explanation ought to serve as a strategy in alleviating the task of the explanation of the meaning. What kind of heuristic assumption is in play when taking such approach? It is assumed that there is a central feature of meaning that can be exploited in such a way that it can be represented by some model. Understanding sentences of some language involves having knowledge of their meaning, and this cognitive feature of meaning is taken to be the central one that can be studied by “a representation of what it is that is known when an individual knows the meaning.”

(2)-(5) gives an account, in the form of one argument, of what kind of knowledge which, *in general*, constitutes the understanding of a language. There are two candidates: explicit and implicit knowledge. (2) gives the definition of the explicit knowledge, and what is further argued in (3)-(5) is that the explicit knowledge of the meaning cannot, *in general*, constitute the understanding of the language.

The idea of the argument (3)-(5) is that explicit knowledge of the meaning requires a mastery of a fairly extensive language, and the problem is to explain how this mastery is acquired without ending in some explanatory infinite regress. The infinite regress consists in the following: an explanation of understanding a language \mathcal{L}_1 via explicit knowledge would in its turn ask for the explanation of understanding the

language \mathcal{L}_2 in which the explicit knowledge is verbalised (formulated), and since, *per ex hypothesis*, understanding consists in explicit knowledge, the understanding of \mathcal{L}_2 is explained by understanding \mathcal{L}_3 , and so on.

There is also a presence of an explanatory vicious circle if we wish to explain how the mastery of one language is acquired without presupposing any knowledge of another languages. The vicious circle consists of the following: (a) the explanation of how understanding of a language is acquired is given in terms of explicit knowledge, but (b) explicit knowledge already presupposes an understanding of some language, an understanding which was sought to be explained in the first place.

The other candidate of the kind of knowledge constituting an understanding of language is the implicit knowledge of meaning. What is the relation between explicit and implicit knowledge? The question is important, because the argument cannot work unless these two candidates exhaust all possibilities by the kind of knowledge constituting an understanding of language. Therefore, in order for the argument to work, implicit knowledge of meaning has to be interpreted as knowledge of meaning that is not explicit, *a fortiori* both explicit and implicit knowledge of meaning exhaust all possibilities relevant for the explanation of meaning. If so interpreted, we have only a negative characterization of implicit knowledge, since it is given only in terms of what it is not.

(6)-(7) are the crucial steps in establishing the conclusion (8). The problem in examining the argument is that we do not have a clear positive characterization of implicit knowledge. It seems that (6)-(7) gives one positive feature of implicit knowledge: the ascription of implicit knowledge to some speaker is legitimate if it is possible to say what the manifestation of the speaker's knowledge consists of. The manifestation in question is understood as something observable in the behaviour of the speaker. However, (8) as a conclusion of (6)-(7) is problematic. (8) maintains that a grasp of the meaning of a statement must, in general, consist of a capacity to use the statement in a certain way, and this use is observable in the behaviour of the speakers of a language. In other words, (8) maintains that a grasp of the meaning of a statement must, in general, consist of a manifested implicit knowledge. However, what we can conclude from (1)-(7) is something weaker than (8), namely that a grasp of the meaning of a statement must, in

general, consist of implicit knowledge. The positive characterization of implicit knowledge, in (6)-(7), does not necessarily give a constitutive feature of implicit knowledge, only a relation between the implicit knowledge of speakers (KS) and the legitimate ascription of implicit knowledge to speakers (LAK). It is the interpretation of the relation between KC and LAK upon which the validity of the argument hinges. (8) follows from (1)-(7) only if the legitimate ascription of implicit knowledge to speakers is a constitutive feature of implicit knowledge. But this is quite problematic to maintain without a further positive clarification of the concept ‘implicit knowledge’, something that is lacking in the argument. Nevertheless, we will discuss this problem when we deal with the relation between Dummett’s three arguments, next we will take a look on the third next argument.

3. The Acquisition Argument

The third argument runs as follows.

[Arg3]

(1) When we learn a mathematical notation, or a mathematical expressions, or, more generally, the language of a mathematical theory, what we learn to do is to make use of the statements of that language: (2) we learn when they may be established by computation, and how to carry out the relevant computations, we learn from what they may be inferred and what may be inferred from them, that is, what rôle they play in mathematical proofs and how they can be applied in extra-mathematical contexts, and perhaps we also learn what plausible arguments can render them probable. (3) These things are all that we are shown when we are learning the meanings of expressions of the language of the mathematical theory in question, because they are all that we can be shown: (4) and, likewise, our proficiency in making the correct use of the statements and expressions of the language is all that others have from which to judge whether or not we have acquired a grasp of their meanings. (5) Hence it can only be in the capacity to make a correct use of the statements of the language that a grasp of their meanings, and those of the symbols and expressions which they contain, can consist. (6) To suppose that there is an ingredient of meaning which transcends the use that is made of that which carries the meaning is to suppose that someone might have learned all that is directly taught when the language of a mathematical theory is taught to him, and might then behave in every way like someone who understood that language, and yet not actually understand it, or understand it only incorrectly. (7) But to suppose this is to make meaning ineffable, that is, in principle incommunicable. (8) If this is possible, then no one individual ever has a guarantee that he is understood by any other individual; for all he knows, or can ever know, everyone else may attach to his words or to the symbols which he employs a meaning quite different from that which he attaches to them. (9) A notion of meaning so private to the individual is one that has become completely irrelevant to mathematics as it is actually practised, namely as a body of theory on which many individuals are corporately engaged, an enquiry within which can communicate his results to others.²⁸

²⁸ Dummett, op. cit., pp. 217-218

The structure of the whole third argument consists of two parts: (1)-(5) and (6)-(9). Actually, the third argument proper is only the first part, while the second part represents the consequences of the conclusion in the first part. The second part of Arg3 is interesting, because it gives an account of the relation between three aspects of meaning: (a) communication of the meaning, (b) manifestation of the meaning, and (c) learning of the meaning. So, if we regard Dummett's third argument only as the first part of Arg3, then the second part of Arg3 is actually a new argument, which connects the previous three arguments as a whole.

The first part of Arg3 is an argument which goal is to establish that a grasp of the meaning of statements in a language *consists in* the capacity to make a correct use of the statements in the language, namely (5), via the idea of learning. The language in question is a mathematical language. (1)-(2) gives an account of how we learn to understand mathematical statements, and (3)-(4) is the point of the whole account, namely that all things involved in learning the meaning of statements are things that can only be *shown* to the speakers of a language. How is (5) established as a conclusion of (1)-(4)?

An important premise is (1): speakers learn mathematical statements if and only if they acquire an ability of how to use the statements. However, (1) is not sufficient in establishing (5) unless there is a necessary feature of learning constituting the grasp of the meaning of statements. The problem of the argument is that this necessary feature of learning is not made explicit in the argument. This implicit idea of necessary feature of learning is what I would call the *idea of learning*, which the argument tacitly presupposes. What does the idea amount to? In answering the question, let us observe what is the point of the argument. The point is, to repeat, that all things involved in learning the meaning of statements are things that can only be *shown* to the speakers of language. Learning involves acquiring a *new* understanding, and precisely giving an account of how to acquire a *new* understanding provides an explanation of what the understanding genetically consists in. It is a genetic account of the meaning exploiting the idea that learning must be manifested by *demonstration*. The genetic account of the meaning follows from general considerations of learning not made explicit in the argument.

Notice the crucial premise in the argument: all things involved in learning the meaning of statements are things that can only be *shown* to the speakers of language. Why is this the case? This question makes the argument problematic, which will be touched upon in the section dealing with the evaluation of the argument.²⁹

4. The relation between the three ideas of arguments

In the interpretation of Arg3, it is understood that the proper part of Arg3 is (1)-(5). It will be referred to as Arg3a. The second part of Arg3 (6)-(9), Arg3b, is rather understood as a corollary of the overall argumentation of Arg1, Arg2 and Arg3a, giving an account in the form of a new argument of how the main ideas of the previous arguments are interrelated for establishing the principle UP; the core principle of the anti-realist challenge.

Support for such interpretation of Arg3 is seen in the observation of how (5) figures in Arg3, namely that it figures as the principal goal to be established by utilizing only the idea of learning, since (5) is the very principle UP.

Although Arg3b is interpreted as a new argument, it should be noted that Arg3b should not be treated as a new independent argument for UP, as it is the case with Arg1, Arg2 and Arg3a. Such treatment would not be consistent with the very semantics of the intuitionistic logic. The strategy of Arg3b is one kind of the *reductio ad absurdum* argumentation that can be valid in intuitionistic logic if it is already established that the disputed supposition, sought to be refuted, is false, where falsity is understood in the intuitionistic sense.³⁰ Hence Arg3b only makes sense if it seen as a corollary of the overall argumentation of Arg1, Arg2 and Arg3b, being dependent of them. The purpose of Arg3b with such *reductio ad absurdum* argumentation is to strengthen the refutation of

²⁹ In the section B4.

³⁰ In the classical case, the goal of *reductio ad absurdum* argumentation is to show the absurdity of supposing some statement's *truth*, a statement sought to be refuted. In the intuitionist case, the goal is to show absurdity in supposing that there is a *proof* of some statement, a statement sought to be refuted (cf. Dummett, *Elements of Intuitionism*, p. 17). The classical *reductio ad absurdum* is based on both the law of double negation and bivalence principle, and it can be valid to employ in the intuitionistic logic *only if* it is applied for effectively decidable statements, where 'decidable' is understood in Dummett's sense, i.e. we have an effective means of which we can recognize that it would yield a proof of the statement (cf. Dummett, *Elements of Intuitionism*, p. 20).

the proposition that “there is an ingredient of meaning which transcends the use that is made of that which carries the meaning.” A refutation of the proposition is an indirect establishment of UP, since UP is the negation of the proposition. If it is not the case that there is an ingredient of meaning which transcends its use, then the meaning of the sentence is exhaustively determined by its use.

What is interesting with Arg3b is how the main ideas of Dummett’s overall justification for intuitionistic logic come into play. If the *learning* of the meaning of mathematical sentences is not *manifested* in the use of it, the meaning of the sentence is in principle *incommunicable* [cf. (6)-(7) of Arg3b]. (8)-(9) in Arg3b serve to elucidate this interplay by emphasizing that the language of mathematics is a public language where a notion of meaning *so* private is completely irrelevant; where “*so* private” is privacy which violates both the idea of *manifestation* and *communication* in the *learning* of new mathematical sentences.

C. Evaluation of Arguments and Their Role in the Formulation of ARC

1. General Observations

What is the common thread in the three arguments pivotal for the justification of intuitionistic logic in Dummett's paper? The common thread in the justification of intuitionistic logic lies in the strategy utilizing the idea of having *evidence for* the presence of some feature relevant in the determination of the meaning of a statement.

The first argument, the Communication Argument, says that a successful communication is possible only if the meaning of a statement is manifested in the use of the statement, where this manifestation in the statement's use is understood as something observable. An important idea in the argument is the idea of a legitimate ascription of meaning to the speaker's statements, where a speaker has the intention to convey the meaning of statements in a communication. A legitimate ascription of meaning to a speaker's statements is conditioned upon the very manifestation of meaning in the use of the statements in a communication where manifestation is observable. Speakers acquire the *evidence for* there being one meaning conveyed in the communication by the observation of the use of statements. The idea of having evidence for there being one meaning communicated between speakers involves that a successful communication is based on a speaker's possession of knowledge of a statement's meaning capable of being manifested in an observable linguistic practice.

Likewise, the second argument, the Manifestation Argument, utilizes the idea of evidence for a speaker's understanding of the meaning of a statement, evidence grounded on an observable manifestation of meaning in the speaker's use of the statement. The idea of evidence comes into play when understanding of the meaning of a statement is treated as *knowledge*.

The attention of the third argument is related to the elaboration of how we learn the meaning of a statement in a language. Although there is no explicit talk about evidence and knowledge, nevertheless, learning is connected to the question of evidence upon which knowledge of meaning is grounded. How are learning, knowledge and evidence related in the argument? In the light of the previous two arguments, the whole gist of the third argument is that learning is acquiring knowledge of the meaning of a

statement. Acquiring knowledge of the meaning of a statement is conditioned upon the observation of how the statement is used in the linguistic practice.³¹ Observations play the role as evidence for understanding the meaning of statement correctly, where this evidence is necessary for learning the meaning of statement.

Unlike the first and the third argument, the second argument directly addresses the question of how do we understand language and meaning of a statement. The argument's premise is that understanding the meaning of a statement should be explained in the terms of knowledge. What kind of knowledge do we talk about in this context? The argument is not so clear about it. It is said that this knowledge is in the form of "representation of what it is known when an individual knows the meaning." This is quite a general characterization of the knowledge pertaining to the understanding of language, and thus not so clear what it could imply. Nevertheless, invoking knowledge is significant, for where there is knowledge there is always talk about evidence. The question of what kind of knowledge is involved in understanding the meaning of a statement is implicitly given in the last argument, where it is mentioned how we learn the meaning of statements, by observing the sentence (2). This is about mathematical statements, where an understanding of their meaning is knowledge of inferential relations between statements. More importantly, it is knowledge about the role they play in mathematical proofs. Implicit in the second and the third argument, when considering them together, is that an understanding of the meaning of mathematical statements is, in an important respect, related to a capacity of forming and recognizing proofs of mathematical statements. If more generalized, including other assertive statements, as for instance empirical statements, an understanding of the meaning is related to a capacity of having an idea of what would count as evidence for a statement.

However, all three arguments, taken together, in spite of what is said above, do not establish that an understanding of the meaning is *consisting of* a capacity of having an idea of what would count as evidence for a statement. Before clarifying this, let also be noted: Even if we would say that they would establish this result, granting that there is no question-begging premises, this would not, *in itself*, represent a justification for

³¹ Note (3) in the third argument: "These things are all that we are shown when we are learning"

discarding classical logic in preference for the intuitionist logic. Recall that, according to Dummett, the only available meaning-theory for a Platonist to accept, in the philosophy of mathematics, is the truth-conditional view of meaning: the meaning of a statement consists in knowledge about truth-conditions, but such that it is not required a recognition of whether these truth-conditions are obtained. This is perfectly consistent with saying that understanding of a statement is conditioned upon having an *idea* of what would count as evidence for the statement, if this *idea* is about knowing what truth-conditions should satisfy in order to have evidence for the statement. It does not tell us what kind of exercisable capacity is involved in order to recognize truth-conditions/evidence. For instance, it does not tell whether we should be capable of producing such evidence. The point is that the very term 'idea', in this context, is obscure and can be interpreted in many various ways, compatible with both semantic realism and semantic anti-realism.

If so, why is it then said that these three arguments are pivotal for the justification of intuitionist logic? The purpose of these arguments is to serve as a first step to the formulation of ARC, namely to establish both UP and $[UP \Rightarrow MP]$. Recall that both UP and MP are premises of ARC. It is ARC, as a whole formulation, which challenges the truth-conditional view of meaning, and not the arguments establishing the premises of ARC. It should also be noted that three arguments for UP and MP give the background for ARC, explaining UP and MP.

We are now interested in evaluating the strength of these three arguments. Are UP and MP established by these arguments?

The general problem with all these arguments is that they at most establish that having evidence for the presence of some (semantic) feature, relevant in the determination of the meaning of a statement, is only evidence for a legitimate ascription of a correct understanding of the meaning of a statement to speakers in a linguistic practice: whether in a communication (1st arg.), or in a context of learning language (3rd arg.), or in a general observable linguistic behavior (2nd arg.). The question is whether this evidence for a legitimate ascription of a correct understanding of a statement's meaning to speakers in a linguistic practice, constitutes the very meaning of the statement. Having evidence for there being an object A is not *necessarily* observing the object A itself. Moreover, evidence for M is not *necessarily* M itself.

Although this general observation of the relation between evidence E for M and M is correct in itself, it does not rule out that E in fact constitutes M. The whole gist of Dummett's arguments is precisely to show that the relation between E and M is such that E constitutes M when dealing with the question of understanding language, where 'M' stands for some linguistic feature relevant in the determination of the meaning of a statement. Let us call this relation between E and M as \mathcal{R} .

I think that only two of Dummett's arguments can give a strong support for \mathcal{R} , namely the first and the third argument. In those two arguments, Dummett focuses our attention on two aspects of linguistic practice: (a) the aspect of communication, and (b) the aspect of learning. Before proceeding with the evaluation of those two arguments, we will again take a look upon the second argument, which I think is the weakest one.

2. The Manifestation Argument (Arg2)

The second argument, namely the Manifestation Argument, suffers from the lack of a clear positive characterization of the implicit knowledge. Dummett introduce the concept of implicit knowledge in terms of what it is not, i.e. giving a negative characterization of the implicit knowledge. However, the negative characterization cannot work in establishing (8), namely that a grasp of the meaning of a statement must, in general, consist of a manifested implicit knowledge. We have also observed that the best interpretation of the argument is to say that (6)-(7) give one positive characterization of implicit knowledge: the ascription of implicit knowledge to some speaker is legitimate if it is possible to say what the manifestation of the speaker's knowledge consists of. However, by such interpretation, we have also seen that the positive characterization of implicit knowledge, in (6)-(7), does not necessarily give a constitutive feature of implicit knowledge, only a relation between implicit knowledge of speakers (KS) and the legitimate ascription of implicit knowledge to speakers (LAK). It is the interpretation of the relation between KC and LAK upon which the validity of the argument hinges. (8) follows from (1)-(7) only if the legitimate ascription of implicit knowledge to speakers is a constitutive feature of implicit knowledge. But this is quite problematic to maintain without a further positive clarification of the concept 'implicit knowledge', something

that lacks in the argument. This problem is more acute by observing the second weakness of Arg2.

The second weakness of the argument is that it does not concretely show what significant features of the linguistic practice connect a speaker's grasp of a statement with the manifestation of the speaker's implicit knowledge. The argument speaks about a manifestation of a speaker's implicit knowledge in general terms, but does not point out the manner of how this manifestation occurs in such a way that it is shown that the manifestation is a constitutive feature of the speaker's grasp of statements. It might be indeed the case that the manifestation of a speaker's implicit knowledge is a constitutive feature of the speaker's grasp of a statement used in a linguistic practice. However, we cannot necessarily establish this as a fact unless we are able to explain how is the manifestation internally connected with a speaker's grasp of a statement.

The other two arguments, in contrast to the Manifestation Argument, focus on an attention either to some feature in a linguistic practice, i.e. communication, or to some form of linguistic practice, i.e. learning language. They look more promising, since in giving an account of these two aspects of linguistic practice we might elucidate how a manifestation of a speaker's grasp of the meaning a statement is internally connected with the speaker's understanding of statements used in practice.

3. The Communication Argument (Arg1)

The goal of the Communication argument is to establish the principle COM, namely that the meaning of a statement consists solely in its role as an instrument of communication between individuals. We have seen that if COM is accepted, then UP follows easily. We have interpreted Dummett's argument as an argument that is only concerned with a public language. The question is whether COM is an acceptable principle for such language. It should be recalled that in the consideration of our question, the issue is not whether a speaker can privately associate content to an expression or statement belonging to a public language. Rather, the issue is whether such a private association may have any bearing in the determination of the meaning of a statement belonging to a public language. The answer is negative in the virtue of violation of rules

determining the correct association of content to a statement. Here follows an argument for the contention.

[Arg4]: A private association of content to an expression is private if and only if an association of content to an expression is governed by the employment of private rules. Rules governing an association of content to an expression are private rules if and only if rules governing an association of content to an expression are not completely constituted by a public agreement on how to associate content to the expression. Rules governing an association of content to an expression are public rules if and only if rules governing an association of content to an expression are constituted only by a public agreement on how to associate content to the expression. An agreement between, at least two, speakers about rules for a correct association of content to an expression is a public agreement on how to associate content to the expression.³² The employment of rules governing the correct association of content to an expression would determine the association as a correct one. Any public agreement on how to associate content to an expression determines rules for employment of correct associations. A language \mathcal{L} is public if and only if there is *both* (1) an expression \underline{s} belonging to \mathcal{L} such that the meaning of \underline{s} is determined only by a set of rules \mathbb{R} governing the correct association of content to \underline{s} , *and* (2) \mathbb{R} can be applied by at least two speakers of \mathcal{L} in some discourse. A set of rules governing the correct association of content to an expression in \mathcal{L} cannot be applied by more than one speaker, unless there is a public agreement on how to associate content to the expression. It follows that, if a set of rules governing the correct association of content to an expression can be applied by at least two speakers in one discourse then the set is constituted by a public agreement, and by that token the set consists by public rules. \mathbb{R} is a set of public rules. Therefore, public rules and not private ones determine meaning of an expression belonging to a public language.

³² The term ‘is’ in the premise serves as a copula and not as the identity operator, i.e. $\forall x (Sx \rightarrow Px)$.

If the above argument is cogent then for any public language, COM is an acceptable principle. However, the above argument is not an interpretation of the Communication Argument, rather it is inspired by Dummett's argument. It does not represent an evaluation of the Communication Argument, rather it represents another support for COM. Therefore, in the evaluation of the Communication Argument, it is instructive to examine how is a communication connected to the question of the relation \mathcal{R} between E and M. If it is shown that \mathcal{R} holds, then the Communication Argument is a strong one. Although **Arg4** is not an evaluation of Dummett's argument, we will see that it is closely connected to the Communication Argument by the examination of \mathcal{R} .

The evidence E for a legitimate ascription of a correct understanding of the meaning of a statement in a communication is simply a speaker's observation of employed public rules belonging to the set \mathbb{R} . These rules determine the meaning in the sense that they govern the correct association of content to a statement. However, these rules governing the correct association of content to a statement are constituted by a public agreement. The public agreement presupposes that a manifestation of a speaker's grasp of the meaning of a statement is possible and observable by all speakers in the context of communication. A speaker's observation is nothing else than the very evidence E, and the linguistic feature M is the set of public rules \mathbb{R} . Since \mathbb{R} is constituted by a public agreement that presupposes E, E is constitutive for the meaning of a statement \underline{s} in \mathcal{L} . Thus, E constitutes M.

4. The Acquisition Argument (Arg3a)

The goal of Arg3a is to establish that a grasp of the meaning of statements in a language *consists in* the capacity to make a correct use of the statements in a mathematical language, namely (5), via the idea of learning. (1)-(4) gives an account of how we learn to understand mathematical statements. How is (5) established as a conclusion of (1)-(4)?

We have seen that (1) is an important premise, namely that speakers learn mathematical statements if and only if they acquire an ability of how to use the

statements. But we have also seen that (1) is not sufficient in establishing (5) unless there is a necessary feature of learning constituting the grasp of the meaning of statements. The problem of the argument is that this necessary feature of learning is not made explicit in the argument. Nevertheless, the argument might work if we interpret Arg3a by assuming that the argument offers an implicit idea, i.e. *the idea of learning*, elucidating how learning constitutes the grasp of the meaning of statements.

The idea of learning involves two things: (a) a genetic account of the meaning,³³ and (b) learning is manifested by a *demonstration*. (a) follows from general considerations of learning, not made explicit in the argument. (b) is implicit in (3), namely that all things involved in learning the meaning of statements are things that can only be *shown* to the speakers of language.

If this interpretation is the best one, there are two kinds of problems with the argument.

The first kind of problem is concerned with the nature of demonstration figuring in the (b), especially if (3) is disputed. It might be asked why is it the case that all things involved in learning the meaning of statements are things that can *only* be shown to the speakers of language. Even if it is granted that learning a language involves certain demonstrations, that there are things that are *shown* in the process of learning, it does not follow that learning is sufficiently constituted by things shown. The argument would be sound if a demonstration is constituted only by observations, where showing is both necessary and sufficient in the process of learning. However, this is quite problematic. Consider one paradigmatic example of learning language, namely the use of ostensive definitions. Since Wittgenstein's *Philosophical Investigations*, many philosophers, as Edward Craig, are accustomed to treating all claims about ostensive definitions with a little caution. Wittgenstein lessons show us that ostensive definitions are not so trivial as they seem *prima facie*, that there is more involved than a mere perceiving an object which is pointed out during a such ostensive demonstration.

[W]e realize that ostension can only succeed in conveying knowledge of meanings with the aid of a considerable contribution from the learner. Out of innumerable features of reality which are

³³ Supra, section B3, p. 26.

presented to him in experience he has the task of selecting the right ones, that is, those which the teacher intends, and as Wittgenstein made it clear, this is never trivial.³⁴

There are two main reasons why ostensive definitions are not so trivial. The first one is that "an ostensive definition can be variously interpreted in every case."³⁵ Wittgenstein's example of an attempt in ostensively defining the number two by pointing to two nuts serves as a fine illustration. In the question of how can two be defined by such ostensive definition, Wittgenstein's interlocutor answers that the learner "will suppose that 'two' is the name of *this* group of nuts!" Wittgenstein's immediate reply is, "he *may* suppose this; but perhaps he does not. (...) And he might equally take the name of a person, of which I give an ostensive definition, as that of colour, of a race, or even a point of the compass."³⁶

The second, and more general one, not concerned with ostensive definition only, is connected with what Wittgenstein says in *Philosophical Remarks*,

in a certain sense, the use of language is something that cannot be taught, i.e. I cannot use language to teach it in the way in which language could be used to teach someone to play piano. – And that of course is just another way of saying: I cannot use language to get outside language.³⁷

As Skorupski notes, any rule or instruction given for the use of a word must be given in language, understood broadly to cover all signs. "Signs can only convey meaning if at some point there is a natural uptake of how they are being used. It is that natural uptake which cannot be taught – it is a condition of the possibility of teaching a language to someone that teacher and pupil share it."³⁸

How are those two above considerations relevant for the evaluation of Arg3a?

The demonstration involved in learning cannot be treated as mere (perceptual) observations of the displayed practice, where the meaning of terms are shown in their

³⁴ Edward Craig, "Meaning, Use and Privacy," in: *Mind* (1982), Vol. 91, p. 542.

³⁵ Wittgenstein, *Philosophical Investigations*, §28.

³⁶ Ibid

³⁷ Wittgenstein, *Philosophical Remarks*, ed. Rush Rhees, trans. By Raymond Hargreaves and Roger White. Blackwell, Oxford, p. 54.

³⁸ Skorupski, *op cit*, p. 31.

use, since the displayed use in itself does not exhibit an intrinsic meaning. The use of the term can be interpreted in various ways, interpretations depending on the learner's hermeneutic horizon. By that token, the manifestation of a statement's meaning in the statement's use cannot be a sufficiently constitutive feature of the process of acquiring *new* understanding. Would this imply that the manifestation of a statement's meaning in the statement's use is not a sufficiently constitutive feature of understanding language? No, not necessarily, since the answer depends on the second problem of the argument.

Also, in connection with the natural-uptake consideration, an understanding of statements depends, in the last instance, of possessing implicit knowledge. By that token, Arg3b inherits difficulties from our evaluation of Arg2.

Up till now, we have considered only the first kind of problems concerning the nature of demonstration involved in the process of learning.

The second kind of problems is concerned with the explanatory value of genetic accounts. Can our explanation of some phenomenon generally be captured by genetic accounts? For instance, if we suppose that a human mind is an emergent phenomenon out of biophysical constitution of human body, and if we furthermore suppose that there is an excellent genetic account of mind's emergence, verified by numerous scientific verifications, would this account necessarily explain the phenomenology of human mind? The point of this question is that it does not follow that a theoretical explanation of how a phenomenon comes into being, generally explains the very phenomenon. So, what we are interested in is the explanatory value of genetic account when it comes to the understanding of language: would an explanation of how we acquire new understanding of concepts also be an appropriate meaning theoretic explanation of what the understanding of language consists in?

I believe that the genetic account of how we learn language has a central place in a full-blooded theory of meaning, since a full-blooded theory of meaning seeks to give an account of how an individual acquires an understanding of some language without presupposing that the individual has any knowledge of other languages. An account of how we acquire understanding of language sheds light on the concept formation and with it the concept possession. However, would such account be an exhaustive account of what the understanding of language consists in? Let us suppose that it does not give an

exhaustive account of concept possession. A meaning theoretic account of features of understanding not captured by the genetic account has then the additional task to explain how these features of understanding are possible. But such additional task is nothing else but a genetic account of these features of understanding. In other words, the meaning theoretic account of understanding cannot ignore the question of genesis of such features. Because of this, a genetic account has a central place in a full-blooded theory of meaning, and any other account must be faithful to a genetic account.

If Dummett's genetic account of the concept acquisition does not provide an exhaustive account of understanding language, would the use theory of meaning be invalidated, and would by this token Dummett's Semantic Anti-Realist Challenge lose its power? Even if a genetic account of understanding cannot give an exhaustive explanation of understanding language, the use theory of meaning is not invalidated if the Communication Argument is accepted. The Communication Argument can be seen as a complimentary account of understanding not captured by the idea of learning.

5. ARC seen in the light of the three arguments

What kind of role do the three arguments play in relation to the formulation of ARC? These arguments play two kinds of roles in Dummett's overall argumentation. The first one is the purely argumentative role in that they serve to establish the underlying principles figuring in the formulation of ARC, namely the Use Principle (UP) and the Manifestation Principle (MP).³⁹ Because of the argumentative role they play in the establishment of the two main principles of ARC, we have been concerned so far with assessing their cogency, i.e. evaluating their soundness as arguments *per se*. However, they play an additional role in Dummett's argumentation. The additional role in question is the elucidative role in that they give the content of the use theory, clarifying the significance of both UP and MP in the formulation of the Challenge.

³⁹ Just to repeat the principles UP and MP as they were formulated in *supra*, section A2, p. 8. UP is the principle maintaining that the use of a sentence exhaustively determines its meaning. MP is the principle maintaining that the knowledge that constitutes a grasp of the meaning of a sentence has to be capable of being manifested in actual linguistic practise.

In reading the formulation of ARC,⁴⁰ we observed that the heart of Dummett's argument is a challenge for a Platonist to show that the Platonist conception does not violate "the principle that use exhaustively determines meaning." It is expected that a Platonist explain what the understanding of the truth-conditions of mathematical statements consists in, when generally those truth-conditions of mathematical statements are not always capable of being recognized as obtaining. However, it is not clear what kind of problems can a Platonist encounter in her attempt to give an explanation in such a way that the explanation would be in harmony with a use theory of meaning. This is so because the term 'use theory' is quite general, having been only characterized by two general principles UP and MP. However, when reading ARC in the light of the three arguments for UP and MP, we get a clearer picture of the kind of problems that a Platonist is challenged to solve.

What kind of problems would a Platonist in mathematics, or a semantic realist, encounter in her attempt to meet the Challenge? The Semantic Anti-Realist Challenge can be formulated in terms of two sub challenges, as Bob Hale does, which sharpens ARC. The sub challenges are known as (1) the Acquisition Challenge, employing the Idea of Learning, and (2) the Manifestation Challenge, employing the idea of the Manifestation Argument.

I find Bob Hale's formulation of the Semantic Anti-Realist Challenge in terms of the Acquisition Challenge and the Manifestation Challenge one of the best interpretation of ARC, and will rather quote both of them:

According to the *Acquisition Challenge*, our training in the use of language consists in our being taught to accept statements as true in circumstances of such-and-such sort, and to reject them as false in circumstances of other sorts. This training *necessarily* proceeds in terms of states of affairs which we can *recognize* as obtaining. But how, in that case, are we supposed to come by the conception of evidence-transcendent truth-conditions which the realist postulates? How are we to come to know what it is for a statement of that kind to be true, or false, in virtue of the obtaining of some state of affairs which obtains *undetectably*?⁴¹

And here is the whole point of the Acquisition Challenge:

⁴⁰ Supra, section A2, p. 4.

⁴¹ Bob Hale, "Realism and its oppositions" in: *A Companion to the Philosophy of Language*, p. 275.

The challenge is to explain how we come to assign to statements truth-conditions involving state of affairs which, by their very nature, *can have played no part* in the process by which the meanings of those statements are learned or communicated.⁴²

The Manifestation Challenges runs similarly to the Manifestation Argument, except for the difference that it also provides both one interpretation of the implicit knowledge and the explanation of the significance of emphasizing it. As in the Manifestation Argument, the first part of the formulation of the Manifestation Challenge argues that knowledge of the meaning of a statement cannot, in general, consist in explicit knowledge. We must also take into account of implicit knowledge of the meaning of a statement.

When knowledge of meaning is not verbalizable but implicit knowledge, it must be knowledge of how to use the sentence, and must thereof consists in the speaker's possession of certain practical abilities. But now, by just what practical abilities is an alleged grasp of evidence-transcendent truth-conditions supposed to be manifested? In the case of effectively decidable statements (...) a speaker's implicit knowledge can be identified with his capacity to discriminate between circumstances in which the statement is true and those in which is not. But it clearly cannot do so in the case of any statement possessed of evidence-transcendent truth-conditions – in this case, there is nothing a speaker can do which fully manifests his supposed grasp of those conditions.⁴³

We see, thus, the elucidating role the arguments figure in the formulation of the Challenge. I think that this elucidating role is the principal objective of the three arguments, namely to elucidate the Challenge which a Platonist appears unable to meet. As such, they do not purport to be conclusive but rather represent a challenge, i.e. a theoretic problem that *prima facie* cannot be solved within a use theory if the Platonist truth-conditional meaning theory is also endorsed.

In the next part, we will consider the Platonist reply to the Challenge. As the chief representative of the Platonist reply, we will consider Alexander George's paper "How not to refute realism", since it both provides a good analysis of the structure of the Semantic Anti-Realist Challenge and distinguishes various kinds of Platonist replies to the Challenge in the philosophy of mathematics.

⁴² Ibid

⁴³ Bob Hale. op. cit., p. 276.

D. The Platonist Reply to ARC

1. Introduction

Up till now we have been concerned whether the premises of ARC, UP and MP, may be established by Dummett's three arguments. This time we will have a different concern about ARC. We will be interested in evaluating the force of ARC but not by questioning the underlying premises of ARC. Rather, we will grant both UP and MP for the sake of argument, and from such a point of view examine whether the intuitionist position follows.

What was the main strategy in establishing the premises of ARC? The strategy was to establish that a legitimate ascription of a correct understanding of the meaning of a statement to speakers in a linguistic practice constitutes the very meaning of the statement. The problem with this approach was the general observation that having evidence for there being an object A is not *necessarily* observing the object A itself. Moreover, evidence for M is not *necessarily* M itself.

Although this general observation of the relation between evidence E for M and M is correct in itself, it does not rule out that E in fact constitutes M. The whole gist of Dummett's arguments is precisely to show that the relation \mathcal{R} between E and M is such that E constitutes M when dealing with the question of understanding language, where 'M' stands for some linguistic feature relevant in the determination of the meaning of a statement.

What would follow if \mathcal{R} holds? Would this be sufficient for establishing the intuitionist anti-realist position? If it is accepted that \mathcal{R} holds, then the principles UP and MP, the premises of ARC, are established, as it was discussed in the section C1. Would acceptance of UP and MP, the heart of ARC, be the ground of justification for the intuitionist position? This depends on the interpretation of E and M, relative of \mathcal{R} .

The relation \mathcal{R} is between E and M, where E is the evidence for a legitimate ascription of a correct understanding of a statement's meaning and M is a semantic/linguistic feature relevant in the determination of the meaning of a statement.

Talk about evidence involves the question about knowledge, and we have noted that mathematical statements are in question here, where an understanding of their meaning is knowledge of inferential relations between statements. More importantly, it is the question of knowledge about their role in mathematical proofs. Implicit in Dummett's argumentation is that an understanding of the meaning of mathematical statements, in an important respect, is related to a capacity of forming and recognizing proofs of mathematical statements. In what important respect is it *related*? The thrust of Dummett's argumentation is that understanding of meaning of mathematical statements is *constrained* within a capacity of forming and recognizing proofs of mathematical statements. It follows that the evidence E for a legitimate ascription of a correct understanding of the meaning of a statement, would only be legitimate if this understanding is constrained within a capacity of forming and recognizing proofs of mathematical statements. If \mathcal{R} holds, what kind of (linguistic/semantic) feature is involved in M? The feature in question involved in human understanding of mathematical discourse is a tacit association with each sentence of proof conditions, where such tacit association be paired with a fully exercisable capacity to recognize that these proof conditions obtain when they do. According to Alexander George, if E and M is so interpreted, then a realist analysis of mathematical discourse would not be a viable one, and the intuitionist analysis would be the preferred one. However, Alexander George's contention is that Dummett's argumentation is a circular one.⁴⁴ The goal of his paper "How not to refute realism" is to show the nature of circularity in Dummett's argumentation. In the next sections, we will follow George's examination of Dummett's antirealist argument.

⁴⁴ Alexander George, "How not to refute realism", in: *The Journal of Philosophy*, vol. XC, No. 2, Feb. 1993

2. Platonist and Intuitionist Correctness Condition Constraints

George's representation of the structure of semantic antirealist argument is as follows:

[Arg5]:

- (1) Human understanding of mathematical discourse consists in a tacit association with each sentence of correctness conditions.
- (2) An important constraint on the choice of these conditions is that each such tacit association be paired with a fully exercisable capacity to recognize that these correctness conditions obtain when they do.
- (3) If we take correctness conditions to be proof conditions (the conditions in which the associated sentence is proved), then the condition of (2) is satisfied.
- (4) If we take correctness conditions to be realist truth-conditions (the conditions in which the associated sentence is true), then the constraint of (2) is violated.
- (5) Therefore, an intuitionist analysis of mathematical discourse (one according to which our understanding consists in tacitly associating proof conditions with sentences) is available, while a realist analysis (one according to which our understanding consists in tacitly associating truth-conditions with sentences) is not.⁴⁵

One might object to the intuitionist's reasoning in at least two ways here. A standard realist objection is to challenge the legitimacy of (2). However, there is a more interesting objection not questioning the legitimacy of (2), but rather questions the premise (4). The antirealist, on this second objection, begs the question in arguing that the realist violates (2)'s constraint. Given an interpretation of (2) acceptable to the realist, (4) is false. Intuitionists conclude that (4) holds on the basis of their interpretation of (2), one that would be rejected by the realist.⁴⁶ Why is the second objection more interesting?

Note that while both the first and second objections to this argument involves charges of *petitio principii*, they should be distinguished. The first rejects (2) outright. A proponent of the second grants it, may even attach formally the same interpretation to it, but charges that settling on a substantive interpretation rich enough to support or reject (3) and (4) involves taking a position on an issue central to realism/antirealism debate. While both objections are important, there is a sense in which the second is more interesting: because it grants more to the antirealist's argument, it reveals more about the argument's weakness.⁴⁷

The correctness conditions figuring in (2) were in the early part of this paper called the correctness condition constraint (CC).⁴⁸ However, CC is not the same as RP.⁴⁹

⁴⁵ Alexander George, op. cit., p. 54.

⁴⁶ George, op. cit., pp. 61-62.

⁴⁷ George, op. cit., p. 62.

⁴⁸ Supra, section A3, p. 13. Just to repeat the formulation of CC; CC is the principle maintaining that an understanding of a mathematical sentence consists in associating correctness conditions for the sentence in

The logical relation between CC and RP is that CC is a more generalized formulation, while RP is an interpretation of CC. To be more precise, CC is logically equivalent with RP only under the interpretation of FEC.⁵⁰ It was also said that the thrust of ARC is that the acceptance of the principle UP will ultimately justify the acceptance of RP. However, a proponent of the second realist objection would say that what UP at most establish is rather CC and not RP. Both UP and the ground for UP, namely the contention that the relation \mathcal{R} holds, could be accepted by such a realist, provided that the relata of \mathcal{R} , E and M, have realist interpretations. The realist interpretation of E and M would respect that the human understanding of the meaning of mathematical statements is *constrained* within a fully exercisable capacity to recognize the correctness conditions for mathematical sentences, but not interpreted that this fully exercisable capacity is the capacity to recognize proofs of mathematical sentences.

George's paper is devoted to explain that it is not possible to demonstrate that RP follows from CC unless there is a presence of some circularity in the demonstration. Therefore, much attention is focused upon the discussion of what the formulation of CC amounts to. The central concept figuring in the formulation is the concept of a fully exercisable capacity to recognize that the correctness conditions of a mathematical sentence obtain when they do. What does it mean for a capacity to be fully exercisable in this context? The answer to the question ought to be as neutral as possible, where both a realist and an intuitionist would agree.

such a way that the association is paired with a fully exercisable capacity to recognize that these correctness conditions obtain when they do.

⁴⁹ Just to repeat the formulation of RP; RP is the principle maintaining that a grasp of the meaning of a statement consists in a capacity to recognize a proof of it when one is presented to us.

⁵⁰ Supra, section A3, p. 13. Just to repeat the formulation of FEC: A fully exercisable capacity to recognize that the correctness conditions for a mathematical sentence obtain when they do is a capacity to recognize a proof of it when one is presented to us.

3. The neutral interpretation of CC and difficulties in the R/AR dispute

In order to arrive at the most neutral interpretation of CC, let us first mention what the concept does not imply. It does not imply speakers to have some effective procedure of how to produce proofs for any mathematical sentence, since both a realist and an intuitionist acknowledge the existence of undecided statements. Furthermore, "given any such [undecided] statement S, intuitionists lack effective means that would eventually place them in circumstances in which they might fully exercise their capacity to recognize a proof of S when presented with one."⁵¹ Therefore, one cannot interpret CC as demanding that each mathematical sentence be correlated with a capacity that speakers can *effectively* exercise, where such exercise requires that we have an effective procedure that would eventually place speakers in circumstances needed for the full exercise of these capacities of recognition. With this observation, George comes with the first important point (P₁) regarding how both a realist and an intuitionist should interpret CC:

[P₁]

If (2) is to function as intended by the intuitionist, the constraint should require only that there be capacities which are correlated with each tacit association of correctness conditions to sentences (...) and which are *possible* for speakers to exercise fully.⁵²

In the evaluation of Arg5, the point P₁ is highly relevant to bear in mind. The importance of George's point lies in highlighting the term 'possibility', which is the central focus in George's continuation. Alexander George shows that the soundness of Arg5 in the last instance depends on how both a realist and an intuitionist conceive this *possibility of exercising the recognitional capacities*. The discussion between a realist and an intuitionist about the soundness of Arg5 has two kinds of difficulties.

The first is connected with the conceptual complexity in the talk about both speakers' recognitional capacities, as figuring in the formulation of CC, and about the possibility of exercising the relevant speakers' recognitional capacities; a complexity introducing important conceptual distinctions bearing on Arg5. In analysing the discussion between a realist and an intuitionist over the soundness of Arg5, it would be

⁵¹ George, op. cit., p. 54.

⁵² George, op. cit., pp. 55-56.

an oversimplification to say that the root of their disagreement is just a matter of utilizing different interpretations of the relevant concepts. Although there is no doubt that both disputants employ different interpretations of the relevant concepts, and that much of their disagreement is due to this difference, both disputants also share some common framework, which will be introduced when taking into account conceptual distinctions in the talk about a speakers' recognitional capacities. Both sides in the dispute may agree with neutral definitions, but disagree with their import. For instance, both would take George's following definition of the possibility of exercising a recognitional capacity as neutral: "a capacity will be possible for speakers to exercise fully if and only if it is possible for them to be in some situation whose obtaining is a precondition for the full exercise of the capacity and also possible for them to exercise that capacity in that situation."⁵³ However, they may disagree about which situations are possible and whether it is possible, in general, to exercise the capacity in question, if this situation is possible. Moreover, due to certain conceptual distinctions in the talk about a speakers' recognitional capacities (see the next section), a realist and an intuitionist may disagree whether some particular distinct capacity is possible to exercise.

The second difficulty is connected to the already observed fact that the discussion may take two different paths, depending on the realist attitude towards the premise (2). To repeat, a realist can either challenge the legitimacy of (2), or not questioning the legitimacy of (2). Although, the second kind of realist attitude is more interesting, we ought to take into account both alternatives in our evaluation of the soundness of Arg5. In both cases, we will observe how the concept of possibility of exercising relevant capacities plays an important role in the dispute. In the first case, the focus is upon the question whether it is, in general, possible for intuitionists to exercise fully their correlated capacities. In this context, a realist would reject the intuitionist claim that it is, in general, possible for intuitionists to exercise fully their correlated capacities. In the second case, the focus is upon the question whether it is, in general, possible for realists to exercise fully their correlated capacities. In this context, an intuitionist would reject the realist claim that it is not generally possible for realists to exercise fully their relevant

⁵³ George, op. cit., p. 56.

recognitional capacities. Of course, in both cases, an intuitionist would reject that it is, in general, possible for realists to exercise their alleged correlated capacities. However, it should be noted what is the difference between these two cases with regard to the effect of the intuitionist contention: the intuitionist contention is not disputable for a realist of the first case, while it is disputable in the second case.

4. Some conceptual distinctions

In the talk about a speakers' recognitional capacities, some conceptual distinctions should be kept in mind. Although George does not make these distinctions explicit in his paper, nevertheless his whole article is hard to follow if these distinctions are not observed:

- 1) 'Correlated recognitional capacity': a capacity correlating each sentence with its correctness conditions. From George's article, this correlation is taken to be based upon some "tacit association with each sentence of correctness conditions" (the first premise of Arg5).
- 2) 'Fully exercisable recognitional capacity': a capacity to recognize that a sentence's correctness conditions obtain when they do. A correlated recognitional capacity is not necessarily a fully exercisable recognitional capacity, but any fully exercisable recognitional capacity is a correlated capacity.
- 3) 'Effective recognitional capacity': a capacity to recognize whether a *presented* construction is a proof or a refutation of a statement (if it is). It is effective in the virtue of possessing an effective method enabling to recognize whether a presented construction is a proof/refutation of a statement. A fully exercisable recognitional capacity is not necessarily an effective recognitional capacity, but any effective recognitional capacity is a fully exercisable recognitional capacity.
- 4) 'Effectively exercisable recognitional capacity': a capacity involving an effective procedure that would enable a speaker to produce a proof/refutation for a statement. Having an effectively exercisable recognitional capacity implies also having an effective procedure that would eventually place a speaker in circumstances needed for the full exercise of correlated recognitional capacities. A fully exercisable recognitional capacity is not necessarily an effectively exercisable recognitional capacity, but any effectively exercisable recognitional

capacity is a fully exercisable recognitional capacity, or at least would lead into making a use of fully exercisable recognitional capacities. An effective recognitional capacity is not necessarily an effectively exercisable recognitional capacity, since we can be presented with some proof/refutation for a statement without having any recipe of how to arrive at such proof/refutation.

Given the above conceptual distinctions, a realist and an intuitionist can have several disagreements in their answer to the following questions:

- a) What are those correctness conditions figuring in the concept of a correlated capacity? For an intuitionist correctness conditions are proof conditions, while for a realist they are truth-conditions.
- b) Is it, in general, possible for realist correlated recognitional capacities to be fully exercised? A realist of the second kind would give a positive answer, while an intuitionist would disagree.
- c) Can intuitionists lay claim even to possible exercise of (intuitionistically constructed) effective recognitional capacities? Both kinds of realists would disagree with the intuitionist's insistence that it is, in general possible, for an effective recognitional capacity to be fully exercised. An intuitionist would insist that it is possible to exercise an effective recognitional capacity even in the case of undecided statements.

The last question is the most interesting in the dispute. The positive answer is in effect nothing else but the affirmation of RP. Both kinds of realists would deny RP, but their motivation for rejecting RP is different. Let us take a closer look at both types of dispute between a realist and an intuitionist, types of disputes depending on the kind of realist rejection of RP.

5. The first type of dispute

In the first type of dispute between a realist and an intuitionist, a realist questions the legitimacy of the second premise. In other words, CC is denied by a realist, and consequently RP would likewise be rejected.

A realist does not dispute the fourth premise of Arg5. A realist agrees with an intuitionist that realist correlated capacities are not in general possible to be fully

exercised. But a realist would hold that intuitionist correlated capacities are in general not possible to be fully exercised either. On the other hand, to repeat, an intuitionist would insist that it is possible for them to exercise fully their correlated capacities, even in the case of undecided statements. Why such disagreement? Their disagreement is not rooted in having different definitions of the possibility of exercising a recognitional capacity, since both agree with the following definition:

A capacity will be possible for speakers to exercise fully if and only if it is possible for them to be in some situation whose obtaining is a precondition for the full exercise of the capacity and also possible for them to exercise that capacity in that situation.⁵⁴

Both sides in the dispute agree with the above neutral definition. Where both sides disagree is about the import of the definition. They disagree about whether certain situations are possible, and whether it is possible, in general, to exercise the capacity in question if a particular situation is possible.

In this kind of dispute, both sides are consistent in their respective positions, and cannot offer objections aiming at the inconsistency of the other side. So, the dispute is not about whether some side is consistent in its position, but about something else. Before clarifying what the dispute is about, let us first examine whether both respective positions are consistent in the dispute.

If Arg5 is viewed as an intuitionist objection aiming at establishing the inconsistency of the realist position, then a realist position is immune against the intuitionist objection even if a realist grants the fourth premise, precisely because a realist is not obliged to accept the second premise of Arg5. After all, "realism ... holds that while all statements are determinately true or false, there might be statements for which there exists ... no substantive proofs or refutations."⁵⁵ Thus, if Arg5 were viewed as an argument aiming at the inconsistency with the realist position, it would be unsuccessful at such a task.

Nor can a realist offer an objection aiming at establishing the inconsistency of the intuitionist position. But a realist might ask: how can an intuitionist be consistent in both

⁵⁴ George, op. cit., p. 56.

⁵⁵ George, op. cit., p. 57.

insisting that it is in general possible for them to exercise fully their correlated capacities and at the same time acknowledge the existence of undecided statements? An intuitionist is consistent because in the case of an undecided statement, an intuitionist holds that "(i) it is possible for a speaker to be presented with a proof or a refutation of it, and (ii) it is possible, upon being so presented, to determine that the construction is indeed a proof or a refutation of the statement."⁵⁶ However, a realist might object here by asking how is it possible to be in such situation of being presented with a proof or of a refutation of an undecided statement. Here follows George's fair account of an intuitionist reply to the realist objection, showing that an intuitionist position is consistent even when considering the case of undecided statements:

Proofs and refutations are ... finite constructions. Any such construction ... is one with which it is possible for humans to be presented. ... For the intuitionist, whether a given statement has a proof is not something that obtains or fails to independently of the mathematician's activity, precisely because proofs are viewed as the products of such activity. ... Furthermore, ... the intuitionist also rejects the view that there is a knowledge-independent and determinate fact of the matter concerning the means (e.g. the conceptual resources) whereby mathematicians can carry out their constructional activity. ... [Also, it should be noted that it is inconsistent within the intuitionist framework to hold] that for some statement we might come to judge that we are incapable of constructing either a proof or a refutation of them. For an intuitionist, to prove that we cannot construct a proof of a statement is just to prove the negation of that statement, that is, to refute it. Consequently, an intuitionist cannot without contradiction entertain the possibility of judging some statement to be both unprovable and irrefutable.⁵⁷

Although both positions in the dispute are consistent, there is a weakness with the realist stance: the realist does not attempt to offer a meaning theoretic alternative to the intuitionist correctness condition constraint of human understanding of mathematical discourse. The premises (1), (2) and (4) of Arg5 should be viewed as a semantic anti-realist challenge, and not necessarily as an intuitionist objection aiming at establishing the inconsistency of the realist position. So, to the question on what the dispute is about, the answer is that it is a semantic anti-realist challenge for a realist to provide a plausible meaning theoretic alternative to the correctness condition constraint unless a realist accepts the correctness condition constraint. Furthermore, our important concern is to evaluate the force of ARC but not by questioning the underlying premises of ARC. Rather, we grant the underlying principles of ARC and from such a point of view

⁵⁶ George, op. cit., p. 56.

⁵⁷ George, op. cit., pp. 57-58.

examine whether the intuitionist position follows. In the context of the discussion about the argument Arg5, CC is seen as the underlying principle of ARC. Therefore, a more interesting realist reply is the one granting the principle CC. The next section deals with the second type of dispute between an intuitionist and a realist, where a realist does accept CC.

6. The second type of dispute

In the second type of dispute between a realist and an intuitionist, a realist does not question the legitimacy of the second premise, and CC is not denied. What a realist denies is the very principle RP.

How is the affirmation of CC and RP related to the premises of Arg5, i.e. how is the realist attitude towards the premises of Arg5?

A realist, as it was observed, does not dispute the first two premises. However, since a realist correctness conditions are truth-conditions, the dispute is about the fourth premise: whether the correctness condition constraint would be violated if the correctness conditions were realist truth-conditions. A realist would deny the fourth premise.

What about the third premise, which states that the correctness condition constraint is not violated if the correctness conditions are proof conditions? A realist would not deny the validity of the conditional of the third premise, when seen from the intuitionist point of view, although a realist would not accept the antecedent of the conditional. Therefore, the premise (3) is not at the issue in the dispute, but, to repeat, the dispute is about the fourth premise.

In order to understand this disagreement about the fourth premise, we should note what is the import of the first two premises of Arg5.

The import of (1) and (2) is about the kind of possibilities of human understanding of mathematical discourse given some proposed interpretation of the correctness conditions of mathematical statements. In the dispute there is a tacit agreement of what any interpretation of the correctness conditions should satisfy. Both a realist and an intuitionist agrees on the following requirement about the interpretation of the correctness conditions:

[REQ]

Any proposed interpretation of the correctness condition constraint should be such that a human understanding of mathematical statements is, in general, possible within the interpreted correctness condition constraint.

Given the above requirement, we readily see what the second dispute between a realist and an intuitionist is about. An intuitionist questions whether the realist interpretation of CC satisfies the requirement REQ, and a realist objective is to meet intuitionist reasons for the fourth premise of Arg5.

What is the principal reason for the intuitionist acceptance of the fourth premise? The intuitionist motive for the fourth premise is connected with the question of infinite domains, the kind of infinity involved in the classical (realist) conception. An intuitionist contends that the realist's correctness conditions, i.e. truth-conditions, cannot be fully exercised by the realist's recognitional capacities, "because of the infinitary nature of certain truth-conditions and the impossibility of our ever taking in such a condition."⁵⁸ According to George's characterization of classical mathematics, "classical mathematics rests on the completed infinite,"⁵⁹ where it is assumed that one can be given all the members of an infinite set, i.e. that all members of an infinite set exist as a comprehensive complete collection. An intuitionist would question whether recognitional capacities are, in general, possible to be fully exercised considering mathematical statements involving infinite domains, but where these infinite domains are understood in the classical/realist sense. George illustrates this problem with the case of Goldbach's conjecture.

Under what conditions does the realist take this statement to be true? Just in case every natural number is Goldbach. According to the realist, this could be so without there being any finitely expressible explanation for why each natural number has this property. The truth conditions of the realist are such that infinite coincidences are intelligible possibilities. If such situation were to obtain, then recognition that it did would require considering the case of each natural number in turn. But this, the intuitionist insists, is obviously impossible. Hence, it is not in general possible for realists to place themselves in a position required for the full exercise of the relevant capacities.⁶⁰

⁵⁸ George, op. cit., p. 60.

⁵⁹ George, op. cit., p. 64.

⁶⁰ George, op. cit., p. 60.

In other words, the intuitionist conclusion, namely that it is not in general possible for realists to place themselves in a position required for the full exercise of the relevant recognitional capacities, is principally based upon (a) the observation of the infinitary nature of the realist truth-conditions, e.g. the truth-condition satisfying Goldbach's conjecture, and (b) the *intuitionist's* conception of what is possible/impossible exercise of the recognitional capacities, e.g. the impossibility of our ever taking in such truth-condition as the truth-condition satisfying Goldbach's conjecture. However, a realist of the second kind, as for instance Alexander George, would maintain that intuitionists are wrong to reach such a conclusion. Although an intuitionist is correct in the observation of (a), (a) is not sufficient for reaching the intuitionist's conclusion. (b) is also required. However, (b) is a question begging premise, based upon the intuitionist conception of possibility/impossibility, and not upon the realist conception. It is a question begging premise, since a realist would ask the following questions:

The argument, we saw, turns on such judgments of possibility. But what kind of judgements are these and what legitimates them?⁶¹

Alexander George, in pursuing the above questions, shows that the kind of possibility in issue is neither a logical nor a physical possibility.

Obviously, *logical* possibility is not what is at the issue. Intuitionists have argued that showing classical mathematics to be consistent would leave their critique untouched: their challenge is not to the consistency but to the intelligibility of that practise. ... Even more clearly, *physical* possibility cannot be what intuitionists have in mind; for on their view it is possible for us to do things that cannot be done in the actual physical world, such as taking in constructions of any finite length and carrying out any finite number of determinate steps in a computation.⁶²

In the intuitionist's view, it is possible for *us* to do things that cannot be done in the actual physical world. This is meant to denote beings with powers that are appropriate idealizations of our powers, otherwise if 'us' denotes humans as they are actually constituted then many capacities the intuitionists will want to attribute is not exercisable by us. Thus, the issue is rather a question of conceptual possibility involved in the judgments of possible/impossible exercise of relevant recognitional capacities, which in its turn involves the question about the legitimate idealizations of our abilities.

⁶¹ George, op. cit., p. 62.

⁶² George, op. cit., pp. 62-63.

We see, then, that what the intuitionist is pointing to is a perceived *conceptual* impossibility basic to the realist position. In order for us to be able to complete an infinite process (e.g., a check or a construction), our actual abilities would have to be idealized in ways that are conceptual incoherent.⁶³

The idealization of our recognitional abilities basic to the realist position are conceptually incoherent, but incoherent from the intuitionist's framework. But, according to George, this does not show whether idealizations disputable for an intuitionist are conceptually incoherent in a realist perspective affirming that infinite processes are completable, for example, that it is possible to collect all the members of an infinite set. From certain realist perspectives, it would be possible to collect all the members of an infinite set "given certain intelligible extrapolations from our actual abilities and from the actual conditions in which those abilities are exercised."⁶⁴

Whether a certain idealization is legitimate, it should be, at least, conceived as an *intelligible* extrapolation from our actual abilities, and if seen from the intuitionist's framework the realist's idealizations are far from being intelligible. If the very idea of a completable infinite process is conceived as incoherent, then any idealization based upon such incoherence will be regarded as illegitimate. To understand why an intuitionist deem the notion of completed infinite as an incoherent one, we should say a few words about the intuitionist notion of infinity, which is called the *potential* infinity. The intuitionist's concept of an infinite set is essentially connected to the idea of an infinite process by which the members of the infinite set is brought into being. The process is infinite only because it cannot possibly come to an end. The idea that an infinite process can come to an end is seen as a highly incoherent one from the intuitionist's framework. Therefore, an

⁶³ George, op. cit., p. 63.

⁶⁴ George, op. cit., p. 65. George mentions different realist's views on the precise nature of these idealizations. Some of these are conceived as taking place either in time or "supertime." For instance, Philip Kitcher's reconstruction of set theory "involves the activity of an ideal mathematical subject whose activity is carried out in a medium *analogous* to time, but far richer than time. (Call it "supertime")." [Kitcher, *The Nature of Mathematical Knowledge*, (New York: Oxford, 1983), pp. 144-146; quoted in George, op. cit., p. 65, n. 18]. Others conceive recognitional processes that do not take place within any kind of temporal order whatsoever. For instance, according to Hao Wang, or at least in Charles Parson's interpretation of Wang's view, "the iterative notion of set involves the process of over-viewing a range of objects, a process that appears not to take place in time at all." [George, op. cit., p. 66, n. 21. See Wang, "The Concept of Set," in *Philosophy of Mathematics: Selected Readings*, (Cambridge University Press, 1983, Second edition), pp. 530-70; p. 531. See also Parson, "What is the Iterative Conception of Set," in *Philosophy of Mathematics*, p. 509].

infinite set is not one where all of its members already exist, but rather is seen as a set whose members cannot all actually exist; "the existence of all but finitely many of them must always remain merely potential."⁶⁵

However, the question remains whether an intuitionist is entitled to insist that the intuitionist's notion of infinity is the only comprehensible conception of infinity. George's point is that the intuitionist's objection against the realist's recognitional capacities, namely that it is not in general possible to fully exercise the realist's correlated recognitional capacities, is based upon "an espousal of the potential infinite as the only comprehensible conception of infinity."⁶⁶ As such, the objection is question begging. Nor can a realist make a case against the intuitionist position if it employs a similar strategy via judgments of what is possible. Why? Because, "judgments about what is possible are so intertwined with those about the infinite that neither can legitimately be taken to provide a basis for the other."⁶⁷

7. Evaluation of George's reply

George's reply is a reply intended to "expose a circularity in an influential argument for antirealism, here understood in the first place as a semantic thesis about the meanings of the sentences of our language."⁶⁸ That the "influential argument" in question is Dummett's semantic objection against the realist position in the philosophy of mathematics can readily be seen from George's first footnote commenting his representation of the structure of the argument; (George's representation of the structure of semantic antirealist argument was introduced with Arg5).

I believe the following [Arg5] closely approximates the argument for antirealism advanced by Dummett and discussed (sometimes adopted, in whole or in part) by many others. I suspect that other arguments for antirealism are closely enough related to this one ...⁶⁹

⁶⁵ George, op. cit., p. 68.

⁶⁶ George, op. cit., p. 68.

⁶⁷ George, op. cit., p. 70.

⁶⁸ George, op. cit., p. 53.

⁶⁹ George, op. cit., p. 53, n. 1.

Arg5 is, then, not an accurate representation of Dummett's semantic objection but represents the structure of the idea of the objection, namely structuring the idea of the semantic anti-realist challenge (ARC). To repeat the formulation of ARC:

[Platonist] conception violates the principle that use exhaustively determines meaning (...) For, if the knowledge that constitutes a grasp of the meaning of a sentence has to be capable of being manifested in actual linguistic practise, it is quite obscure in what knowledge of the condition under which a sentence is true can consists, when that condition is not one which is always capable of being recognized as obtaining.⁷⁰

The key idea is the problem of knowledge of correctness conditions when these correctness conditions are not always capable of being recognized as obtaining. Therefore, the key concept figuring in Arg5 is the concept of correlated recognitional capacity: a capacity correlating each sentence with its correctness conditions. The first two premises in Arg5 serve to introduce the concept of correlated recognitional capacity, and the premises (3) and (4) are George's reformulation of ARC using the concept of correlated recognitional capacity. Relating to the question whether George adequately interprets Dummett's argument, I do not see any problem with Arg5 as an adequate interpretation of Dummett's argument.

George's principal strategy in showing the question-begging character of the intuitionist semantic objection is to uncover one intuitionist tacit assumption. What assumption do we talk about? It is the meaning theoretic assumption about the grasp of universally quantified statements, i.e. where the universal quantifier is employed. The problem of how we understand such statements becomes acute when dealing with statements involving quantification of either finite but unsurveyable domains or infinite (denumerable) domains. George concentrates on the question of statements involving infinite domains, e.g. Goldbach's conjecture. Therefore, his article mostly deals with the difference between the intuitionist and Platonist notions of infinity, and more importantly with the intelligibility of the disputed Platonist notion. The point is that there is a suspicion whether the semantic intuitionist objection tacitly employs the notion of potential infinity. George's objective is to demonstrate that the semantic antirealist objection is heavily based upon accepting the potential infinity as the only intelligible

⁷⁰ Dummett, "The Philosophical Basis of Intuitionistic Logic" in: *Truth and Other Enigmas*, p. 224

notion of infinity. It is precisely this tacit intuitionist assumption that is sought to be uncovered in George's reply.

Reading George's article, the dispute is then reduced to the question of the relation between the notion of potential infinity and the overall structure of anti-realist objection seen in Dummett's arguments. Is Dummett's objection infected by any intuitionist question-begging assumption that already takes a stand on the question of which notion of infinity is the legitimate one? Is the intuitionist notion of infinity, i.e. the potential infinity, rather a consequence of the argument without the involvement of *petitio principii*? George's answer is that the intuitionist account cannot be a necessary consequent of the argument because a realist meaning theoretic account could be compatible with Arg5 when seen from the realist point of view.

I think that George's focus upon the question of infinity is appropriate in the general dispute about semantic realism versus antirealism, because an understanding of logical constants, or more generally semantics of logic, is intertwined with the understanding of infinite domains. Consider the sentence P.

P: $\forall x \in D, F(x)$.

P is equivalent with Q.

Q: for every member \bar{a} of D, $F(a_1) \wedge F(a_2) \wedge \dots \wedge F(a_k) \wedge \dots$.

Every conjunct $F(\bar{a})$ is such that every member of D is an argument of the predicate F.

If the domain consists of finite objects, then we must determine whether for each member \bar{a} of D, $F(\bar{a})$ is true. Precisely this kind of determination involves an understanding of the concept of totality, which is a notion of conceiving every member of some domain by taking into account each member of the domain. The problem emerges when we deal with statements involving quantification of domains with infinite objects. Also in such a case, we must determine for each member \bar{a} of D whether $F(\bar{a})$ is true.

We observe that an understanding of some logical constants, as the universal quantifier, depends on an understanding of the notion of totality and/or infinity. We might understand totality and/or infinity in two exhaustive ways: *either* as the understanding of totality and/or infinity legitimizing the notion of completed infinity, *or* as the

understanding of totality and/or infinity rendering the notion of completed infinity as illegitimate.

Since an understanding of logical constants depends on the question of the legitimacy of the notion of completed infinity, George's discussion about the question of infinity is appropriate in the general dispute about semantic realism versus semantic antirealism. However, is George's focus upon the question of the infinity appropriate when discussing particularly Dummett's arguments for the intuitionist semantics of logic? Dummett's philosophical justification for the intuitionist logic, as seen in the three arguments for the premises of the ARC, is based upon general meaning theoretic considerations not touched by George's reply. Even if we grant that an intuitionist is guilty of the charge of *petitio principii* in Arg5, neither the second type of the Platonist realist reply provides a meaning theoretic account of evidence-transcendent truth-conditions, as paradigmatically exemplified in the undecidable mathematical statements involving infinite domains. What is at most established by the Platonist reply is that some Platonist conceptions involving evidence-transcendent truth-conditions, as the concept of completed infinity, are coherent within various Platonist frameworks in the philosophy of mathematics.⁷¹ It is not clear whether such conceptions are legitimate even in meaning theories where the neutral principle CC is acknowledged. However, when speaking about the legitimacy of some concept, then the intuitionist focus, in the context of discussion of Arg5, is not about which possibilities are conceivable, but rather with providing a meaning theoretic account of how we understand such envisaged possibilities. The challenge in Arg5 is that a Platonist in the philosophy of mathematics is unable to provide a meaning theoretic account of what the understanding of the realist truth-conditions consists in.

However, it might be objected to my contention by pointing out that George in fact meets the Challenge of ARC. The objection runs as follows:

Although George does not give a general meaning theoretic account that meets the Challenge of ARC, George gives at least an account of understanding of one fundamental concept of classical mathematics, namely the classical concept of

⁷¹ See the footnote 64, *supra*, p. 55.

completed infinity. The account employs the idea of enhanced recognitional abilities, or idealization of our recognitional abilities, but such that are legitimate. An idealization is legitimate if it obeys the principle CC, which is interpreted in such a way that the requirement REQ is satisfied.

The above realist objection is not quite correct. What George at most does is to provide a possible route to such an account. The route is in employing the idea of enhanced recognitional capacities. George mentions various Platonist idealizations of human recognitional capacities,⁷² but it is not clear whether these idealizations are legitimate in the sense that they satisfy REQ. The question of the legitimacy of these idealizations is quite a complicated issue, which cannot be dealt with in this thesis. Nevertheless, one problem about the legitimate idealizations of recognitional capacities that will be discussed, is whether a Platonist idealization of recognitional capacities satisfying the requirement REQ has the kind of legitimacy relevant in meeting the Challenge of ARC. To repeat, “when speaking about the legitimacy of some concept, then the intuitionist focus, in the context of discussion of Arg5, is not about which possibilities are conceivable, but rather with providing a meaning theoretic account of how we understand such envisaged possibilities.”

Is the route to a realist meaning theoretic account via the idea of enhanced recognitional abilities a promising one? Bob Hale’s reply to the question is as follows:

[I]f the suggestion is to serve the realist’s ends, it must go beyond envisaging relatively uncontroversial, finite extensions of our detective abilities, to encompass conceiving, for example of creatures capable of surveying infinite totalities, or ‘directly seeing’ into the past and future, or into the minds of others. It is not, however, at all clear that we can conceive any such thing in relevant detail – it is one thing to appeal to the idea of creatures whose recognitional capacities *finitely* extend powers we *actually* possess, and quite another to claim that we can conceive of creatures endowed with capacities for which we have *no* actual model, or which constitute *infinite* extension of our capabilities.⁷³

“If we cannot conceive any such thing in relevant detail” how can we then arrive at a meaning theoretic account of understanding mathematical statements possessing evidence-transcendent truth-conditions via employment of the idea of unconceivable

⁷² Ibid

⁷³ Bob Hale, op. cit., p. 278.

recognitional capacities? To understand this problem, let us take the example of the notion of divine omniscience. In this context, it is worth mentioning Dummett's discussion about the relation between theism and semantic anti-realism. Dummett comes with three points relevant for our discussion about idealized recognitional abilities. I will highlight these points by enumerating them in the quotation of Dummett's discussion.

I have heard it maintained that this [verificationism] is an atheistic doctrine, on the ground that God, not being subject to our limitations, must know of every proposition whether it is true or false, so that our inability to determine this should not lead a theist to doubt bivalence. This argument begs the question by assuming that every proposition *is* either true or false; [1] God's omniscience involves that he knows every true proposition, but it says nothing about how many true propositions there are. (...) [2] The appeal to God's knowledge in no way serves to explain in what our knowledge of the conditions for our statement to be true consists, if there is no explanation of it without that appeal. (...) [3] Granted, God's thoughts are not as our thoughts, but that seems beside the point; we are concerned not with divine concepts but with God's knowledge of truths involving human concepts.⁷⁴

The first point is that the traditional notion of God's omniscience can be compatible with the rejection of the principle of bivalence. Furthermore, when appealing to God's omniscience, we must also be clear about the fact that there are several theological models of omniscience that would deny that God's knowledge involves certain classes of statements, as for instance the class of statements involving future events, chiefly because it is held that the principle of bivalence does not apply to these kind of statements.⁷⁵ The point is that as there are disputes about the concepts of infinity, so are there disputes about the concepts of omniscience. Which leads us to the second point.

The second point is that appealing to the idealized recognitional abilities would not help since it inherits the same problems that are present in giving an account of understanding of statements possessing the evidence-transcendent truth-conditions, e.g. undecidable statements involving infinite domains. The problem would then be re-formulated as the problem of the intelligibility of idealized recognitional abilities. It is not clear how this new problem can be solved if the problem of the legitimacy of the notion

⁷⁴ Michael Dummett, *The Logical Basis of Metaphysics*, pp. 318-319, 349.

⁷⁵ Christian theological models of omniscience that can be compatible with semantic anti-realism are within the framework of Neo-Arminian theology and the Open View of God. Known theologians and Christian philosophers endorsing such models of omniscience are Lorenzo Dow McCabe, J. R. Lucas, Clark Pinnock, and Gregory Boyd.

of completed infinity is unable to be solved without appealing to the idealized recognitional capabilities. As Brian Loar observes,

The problem is that any such superhuman verification procedure, such as counting an infinite set, is such that its implementation by a superhuman creature would be as verification-transcendent for us as the state of affairs whose verification procedure it is supposed to be. So the conception of such procedure should be as problematic as the conception of the corresponding state of affairs.⁷⁶

It is problematic not in the sense that we deny the possibility of superhuman verification procedures, but rather in the sense that it is problematic as a heuristic device in the explanation of *human* understanding of the verification-transcendent truth-conditions. The issue is not whether evidence-transcendent state of affairs are possible but rather whether it is possible to provide an explanation of human understanding of the meaning of statements involving verification-transcendent state of affairs in the terms of truth-conditions. Given that we understand a mathematical statement for which we have not proof, nor a proof of its negation, the problem is to explain what does the understanding of such a statement consist of. If there is a problem in giving such explanation for *human* understanding, what could we say about giving an explanation of superhuman understanding? An appeal to idealized recognitional abilities would in its turn inherit the same problems we encounter for solving the meaning theoretic problem for human understanding.

The third point is about the problem of relevance: what relevance does the appeal to idealized recognitional capacities have for the issue when the dispute is about giving the account of *human* understanding of mathematical statements?

[E]ven if we could imagine a use for certain sentences, by beings with powers greatly exceeding our own, this does not automatically put us in a position to use those sentences in that way ourselves. To suppose that we could is to suppose that we could employ those sentences with the intention of conforming to standards of correct use, even though we are ourselves entirely unable to tell whether our use accords with them or not. This seems to run afoul of considerations concerning the *normativity* of meaning.⁷⁷

The normativity of meaning is about the rules discriminating between correct and incorrect use of statements in language. Bob Hale's point with the normativity of

⁷⁶ Brian Loar, "Truth Beyond All Verification," p. 90

⁷⁷ Bob Hale, op. cit., p. 279.

meaning is that the normative dimension of human language does not encompass the use of sentences requiring recognitional capacities transcending human actual recognitional capacities. This point further strengthens the point with the previous paragraph: any attempt to explain what the understanding of evidence-transcendent truth-conditions of a statement consists of would be doomed to failure if the explanation appeals to idealized recognitional capacities not encompassed by the normative dimension of human discriminating powers between correct and incorrect use of statements. Therefore, to paraphrase Dummett's remark about God's omniscience, we are concerned not with superhuman concepts but with superhuman knowledge of truths involving human concepts.

If we interpret George's discussion about legitimate idealizations of recognitional capacities as meant as a meaning theoretic reply to the Challenge of ARC, then such meaning theoretic reply is threatened by the above anti-realist objections, objections not discussed in his paper.

The fact that the anti-realist objections are not discussed in his paper points out the weakness of the interpretation expressed by the objection to my contention. George's considerations about legitimate idealizations have rather another objective than the meaning theoretic one. His point is that Platonist idealizations of recognitional capacities are legitimate from a realist point of view. However, it is important to understand in which sense they are legitimate. They are legitimate in the sense that they meet the requirement of REQ. This, however, would not imply that the Challenge of ARC is met. The requirement of REQ does not say anything about giving an account of what an understanding of evidence-transcendent truth-conditions consists in, but merely that the idealization in question does not violate the principle CC. George is correct when he points out that an intuitionist cannot object to Platonist idealizations of recognitional capacities, as idealizations of an understanding of mathematical statements involving infinite domains, if such an objection is based upon judgments of what is possible. Such kind of objection would be certainly a question begging one, because "judgments about

what is possible are so intertwined with those about the infinite that neither can legitimately be taken to provide a basis for the other."⁷⁸

Nevertheless, the target of ARC is neither Platonist envisaged possibilities nor the possibility of understanding such envisaged possibilities, but rather the Platonist meaning theoretic *account* of understanding these possibilities, an account which is expected to answer the question: what does the understanding of these possibilities consist of?

⁷⁸ George, op. cit., p. 70.

E. The Holistic Realist Reply to ARC

1. The general meaning theoretic thrust of ARC

In the previous chapter of the thesis, we have been concerned with a realist reply restricted only to the dispute between a Platonist and an Intuitionist. The common meaning theoretic framework of the dispute is the compositional meaning theoretic framework. Both an Intuitionist and a Platonist hold that the application of the predicate ‘true’ to a sentence must be explained in the terms of composition of a mathematical sentence.

However, Dummett’s philosophical justification for intuitionistic logic is based on some general meaning theoretic considerations. Given the general thrust of Dummett’s argumentation, the argumentation preceding ARC, ARC cannot be restricted only to the context of the dispute between Intuitionists and Platonists concerning the relation between truth and verifiability of mathematical statements. A non-compositional meaning theoretic approach and other classes of statements, as empirical ones, should also be considered in the evaluation of the strength of ARC. Dummett is clear about the problem.

[T]he form of argument (...) is virtually independent of any considerations relating specifically to the *mathematical* character of the statements under discussion. The argument involved only certain considerations within the theory of meaning of a high degree of generality, and could, therefore, just as well have been applied to any statements whatever, in whatever area of language. (...) What is involved is a thesis in the theory of meaning of the highest possible level of generality. Such a thesis is vulnerable in many places: if it should prove that it cannot be coherently applied to any region of discourse, to any class of statements, then the thesis cannot be generally true, and the general argument in favour of it must be fallacious. Construed in this way, therefore, a position in the philosophy of mathematics will be capable of being undermined by considerations which have nothing directly to do with mathematics at all.⁷⁹

How are Dummett’s meaning theoretic considerations general in nature? His account of the language is general because it is based on one general principle, namely the principle UP. Let us call any meaning theory faithful to the principle UP as *a use theory of meaning*. The question is whether some other use theory of meaning, which is not a compositional verificationist theory, can be a semantic realist one. One candidate is a holistic theory introduced by Brian Loar. His article “Truth Beyond All Verification”

⁷⁹ Dummett, “The Philosophical Basis of Intuitionistic Logic”, pp. 226-227

represents one possible reply meeting the Challenge of ARC, which will be considered in the present chapter.

2. Truth in Loar's Holistic Theory

In his early days, Dummett characterized holism as a doctrine which is holding that a sentence does not have an individual content.

On a holistic view, no mathematical sentence, nor even an entire mathematical theory, has any significance on its own: it has significance only as forming part of other theories, particularly scientific theories, which can be judged correct or incorrect on the basis of experience, but, again, only as whole.⁸⁰

However, in his later works, Dummett concedes that such a characterization of holism is not a satisfactory one. For instance, in *The Logical Basis of Metaphysics*, Dummett says:

It is tempting to describe holism as the doctrine that a sentence does not have an individual content; but that is unsatisfactory. (...) Holism is better characterized as the doctrine that the application of the predicate 'true' to a sentence cannot be explained in terms of composition.⁸¹

How is the application of the predicate 'true' to a sentence explained in Brian Loar's holistic theory which objective is to address the Challenge of ARC? The predicate 'true' is explained in terms of the disquotational truth schema, treating the truth-predicate as a disquotational truth-predicate where there is no commitment to some substantial notion of truth.

The disquotational truth schema is a device in the analysis of true sentences of some object language \mathcal{L} not containing the predicate 'true'. The predicate 'true' rather belongs to the metalanguage used in the study of the object language \mathcal{L} , i.e. the disquotational truth-predicate figuring in the schema is a metalinguistic predicate. For instance, a sentence "snow is white" belonging to the language \mathcal{L} , where \mathcal{L} is a fragment language of English, is specified to be true in \mathcal{L} by the T-sentence T_s .

T_s : "Snow is white" is true if and only if snow is white.

⁸⁰ Dummett, *The Elements of Intuitionism*, p. 3

⁸¹ Dummett, *The Logical Basis*, p. 230-231

The above T-sentence does not belong to the object language but to the metalanguage, (which in this case is English not limited to the fragment language). The truth of the sentences of an object language is specified by the T-sentences of the metalanguage. Truth is, thus seen, as relative to the object language, where truth is understood as truth-in-L.

However, a theory of meaning where the predicate ‘true’ is explained in terms of the disquotational truth schema is not thereby characterized as a non-compositional theory of meaning. It depends on how the specification of true sentences is determined. For instance, a theory of meaning employing Tarski’s semantic theory of truth would be a compositional theory of meaning. In Tarski’s semantic theory, the predicate ‘truth’ is defined by *recursion* of truth-in-L. A recursive definition of truth-in-L is formulated in an axiomatic manner by the analysis of the compositional structure of the sentences of \mathcal{L} .

How is the specification of true sentences determined in Brian Loar’s holistic theory where truth is understood as purely disquotational truth? Brian Loar characterizes his holistic theory as the two-component theory.

The two-component theory of meaning is, I believe, correct; and as a theory of meaning, it may be shared by proponents (...) of [my] reply.⁸²

The two-component theory distinguishes two aspects of the meaning: (1) the conceptual roles of sentences, and (2) the truth-conditions of sentences.

Brian Loar understands the conceptual roles of sentences as follows.

The conceptual roles of sentences include recognitional abilities with which they are associated, inferential dispositions, and the background of “theories” whose acceptance affects those recognitional abilities and inferential dispositions. The verification conditions of a sentence would of course be part of its conceptual role in this sense; in general, the notion of a use theory of meaning is intended to cover a conceptual role semantics.⁸³

The other aspect of the meaning, i.e. truth-conditions, is “determined by the extrinsic relations of our thoughts or sentences to external things, properties, and state of

⁸² Brian Loar, “Truth Beyond All Verification”, p. 92

⁸³ Brian Loar, op. cit., p. 91

affairs.”⁸⁴ To answer the question how the disquotational truth of a sentence is determined, we should note how the two components of meaning in Loar’s holistic theory are related.

On the two-component model, the theory of understanding or conception is only part of the theory of meaning; it concerns how thoughts or sentential acceptances are related to each other, to perception and to action. But the theory of truth-conditions and reference is concerned with patterns of relations between sentential acceptance or thoughts, as individuated by the theory of understanding or conceptions, and external state of affairs, relations that have to do with success of speakers or thinkers in representing those state of affairs. Those representational capacities are *constituted* by patterns of correlations between thoughts or sentences and extrinsic state of affairs – think of how the numbers of rings in a tree represents its age.⁸⁵

We see, thus, that the truth-predicate points to the relation between language and the world. However, this would not necessarily involve the notion of truth as understood by the correspondence theory. In the correspondence theory of truth, the truth of a sentence is understood as *constituted by* the correspondence relation between the sentence affirming that a particular state of affairs obtains and the fact that the state of affairs in question obtains. The correspondence theory, as well as the theories as the coherence and pragmatic theories of truth, tells what the truth of a sentence consists of, or, in other words, exhibits the essential nature of truth. The correspondence theory does not merely state an analytic definition of truth, in the form of some biconditional, providing a *criterion* of the application of the truth-predicate, but speaks of truth as a *property*.⁸⁶

What kind of relation is involved between language and the world in Brian Loar’s theory when speaking of the disquotational truth-predicate?

[T]he disquotational reference relations and truth predicates that one applies to one’s own sentences do involve genuine *relations* between words and things; that that is so does not require semantic realism. “‘Paris’ denotes Paris” associates a name with a city. To say that ‘denotes’ is used merely disquotationally is, in effect, to say that for a definition to be adequate it must merely pair certain expressions with certain things. Such a pairing can be achieved enumeratively for names; as regards that pairing of sentences with possible state of affairs (so to speak) which one’s disquotational ‘true’ captures, a recursive definition would be required to explicitly capture that truth predicate, although one has effortlessly mastered it without benefit of Tarski. The point is

⁸⁴ Ibid

⁸⁵ Brian Loar, op. cit., pp. 91-92

⁸⁶ See more on the distinction between metaphysical and analytical accounts of truth in: Ralph C. S. Walker, “Theories of truth”, in *A Companion to the Philosophy of language*, pp. 311-314.

that such a definition would associate expressions with, as one must see them, non-linguistic matters.⁸⁷

The problem with Loar's above account of the disquotational truth-predicate is that his theory does not appear to be holistic, as in Dummett's understanding of the term 'holism'.⁸⁸ In the above account, the disquotational truth-predicate is said to be defined by *recursive* definition, starting from the definition of terms, as 'Paris', to sentences, like, "Paris is crowded." Generally, a recursive definition of an expression proceeds by first specifying a special subclass of the items it applies to. Next, we proceed to specifying how the expression applies to remaining items in terms of a relation, such that any new item is related to an item to which the expression already applies. Since the recursive relation in question is with respect to sentences of some language, it is unclear how is the recursive relation between sentences specified without considering the compositional structure of sentences. The disquotational truth schema is usually employed in formal languages, and in such formal context, the compositional structure of sentences is not ignored. When applying the disquotational analysis of truth to natural languages, formal languages serve as a model in such task, where Tarski's semantic theory of truth is a paradigmatic example.

Therefore, the most plausible interpretation of Loar's holism would be that it allows an explanation of the predicate 'true' to a sentence in terms of the composition of the sentence. Such interpretation is confirmed by Loar's reply to Dummett's characterization of holism.

Dummett has characterized holism as "the doctrine that the application of the predicate 'true' to a sentence cannot be explained in terms of the composition of the sentence." Consider again both the disquotational 'true' and the interpersonal 'true in \underline{x} 's language'. Nothing in a holistic theory of one's understanding of 'Paris is crowded' precludes one's judging that 'Paris is crowded' is true iff the referent of 'Paris' is in the extension of 'crowded', or making the corresponding componential judgment about the extrinsic truth-conditions of that sentence in another person's language.⁸⁹

⁸⁷ Brian Loar, op. cit., p. 95.

⁸⁸ To repeat Dummett's characterization of holism. Holism is the doctrine that the application of the predicate 'true to a sentence cannot be explained in terms of the composition of the sentence. See supra, p. 66.

⁸⁹ Loar, op. cit., p. 111

Loar's reply is in the context of meeting Dummett's objections against holism (section VII, pp. 108-111). If Loar agrees with Dummett's characterization of holism then there is expected an explanation how is the disquotational predicate recursively defined without considering the compositional structure of sentences. Such explanation is lacking in the above reply, rendering Loar's reply very obscure. This points out that we should rather interpret Loar as disagreeing with Dummett's characterization of holism.

Nevertheless, Loar is very insistent on explicating truth as a disquotational truth. What is the motivation for such insistence in the context of the dispute on realism versus anti-realism? The answer is connected to Loar's view about realism and to the question of what kind of notion of truth realism is committed to. This will be the topic of the next section.

3. Realism and Truth

In the context of the discussion about what kind of notion of truth realism is committed to, Loar is contrasting his view of realism with Dummett's view.

Dummett takes the realist to be committed to some *substantive* notion of truth, over and above the purely disquotational 'true', that is, the intralinguistic 'true' such that 's is true' is a metalinguistic variant of s. But I do not think that realism, as characterized so far, needs more than the disquotational 'true'.⁹⁰

The reason behind Dummett's view that a realist is committed to some substantive notion of truth lies in Dummett's characterization of various R/AR disputes. A standard characterization of R/AR disputes is that they concern the existence of some sort. For instance, in philosophy of mathematics the dispute is whether mathematical objects exist independently of mathematical activity constructing them, whether they are mind-independent objects, and similar. In philosophy of mind, the dispute is whether mental properties are independent of physical properties. However, according to Dummett, R/AR disputes are best characterized not as disputes about the existence of entities of some problematic sort, but in terms of a certain class of statements. He calls such a class as the disputed class. What is disputed is not whether statements of the disputed class are true, since an anti-realist will not question whether they are true, i.e. an anti-realist can readily

⁹⁰ Loar, op. cit., p. 83

agree that the statements of the disputed class are in many cases true, but rather the character of the notion of truth which may be applied to them.

Realism I characterize as the belief that statements of the disputed class possess an objective truth-value, independently of our means of knowing it: they are true or false in virtue of a reality existing independently of us. The anti-realist opposes to this the view that statements of the disputed class are to be understood only by reference to the sort of thing which we count as evidence for a statement of that class. (...) The dispute thus concerns the notion of truth appropriate for statements of the disputed class; and this means that it is a dispute concerning the kind of *meaning* which these statements have.⁹¹

We see that Dummett takes the realist to be committed to some substantive notion of truth, but in the sense “that statements of the disputed class possess an objective truth-value, independently of our means of knowing it.” However, saying that a statement possesses an objective truth-value would not imply that the predicate ‘true’ is understood as over and above the disquotational ‘true’.

The objectivity of a truth-value involves a relation between the truth-value of a statement and the possession of evidence for the statement, such that the truth-value is considered as independent of our possession of the evidence for the statement. One could ask, why is it independent? It is independent because the statement is “true or false in virtue of a reality existing independently of us”, and consequently the statement is true or false independently of our possession of evidence supporting or refuting the statement in question. This independency relation makes the truth-value objective. Such understanding of ‘objective truth-value’ has not necessarily an ontological import postulating that truth is some kind of real property à la universals. Furthermore, by saying that statements “are true or false in virtue of a reality existing independently of us”, it does not tell how this independent reality renders statements as true or false; only that there is some genuine relation involved between the independent reality and our language captured by the predicate ‘true’.

However, when Loar emphasizes that a realist is not committed to some substantive notion of truth, the term ‘substantive notion of truth’ is not understood in the terms of objectivity of the truth-value of a statement. In Loar’s usage of the term, a substantive notion of truth is an understanding of the truth-predicate “over and above the

⁹¹ Dummett, “Realism”, in: *Truth and Other Enigmas*, p. 146

purely disquotational ‘true’.” In other words, a substantive notion of truth denies that the disquotational schema, or its instances, amounts to a full account of the truth-predicate.

Because of such a terminological difference in the usage of the term ‘substantive notion of truth’, it is not clear whether Loar would disagree with Dummett’s characterization of realism as a commitment to the notion of an objective truth-value. In Loar’s account of the disquotational truth-predicate, we observe, “the disquotational reference relations and truth predicates that one applies to one’s own sentences do involve genuine *relations* between words and things.” In the same vein, it is said:

It seems sometimes to be thought that were the disquotational ‘true’ the only legitimate conception of truth, that would in itself have anti-realist consequences, as though it kept reality at the level, so to speak, of thoughts and sentences. But the right hand side of ‘is true iff’ is not in general about thoughts or sentences. The disquotational ‘true’ may be seen as a trim way of speaking of the obtaining of states of affairs: to say ‘snow is white’ is true is to say in effect that the state of affairs that snow is white obtains.⁹²

Dummett and Loar would agree that realism is committed to the notion of truth transcending verifiability, and in such a sense, a realist is committed to a substantive notion of truth.

The disquotational ‘true’ is our device for generalizing such assertions: “for all \underline{s} of such and such class, it could happen that \underline{s} were true although \underline{s} were unverifiable.” In a certain sense, of course, this does commit the realist to a substantive notion of truth, for it implies that truth can transcend verifiability. But that does not prevent it from being the disquotational ‘true’.⁹³

What is Loar’s point with emphasizing that realism is not committed to the notion of truth, over and above the purely disquotational ‘true’ in the context of the dispute on realism versus anti-realism? The point is that it is a denial of Dummett’s fundamental assumption about the meaning theoretic framework of realism, according to which a realist conception of meaning and understanding has to be framed in terms of truth-conditions. The adherence to a disquotational notion of truth is a kind of deflationary theory of truth, and deflationism about truth entails that meaning cannot be explained in terms of truth-conditions. Before taking a closer look at how deflationism about truth

⁹² Loar, op. cit., p 84

⁹³ Loar, op. cit., p. 83

entails that meaning cannot be explained in terms of truth-conditions (in sect. E5a), we should note the significance it plays in Loar's reply to ARC.

The Challenge of ARC is to explain what the understanding of the truth-conditions of statements consists in, when these truth-conditions of statements generally are not always capable of being recognized as obtaining. However, if a realist understanding is not a matter of grasping verification-transcendent truth-conditions, but in something else, the Challenge is met. "This something else" is knowledge of how to use statements. Loar's article is devoted to show that a realist meaning theoretic framework can be faithful to the principles of a use theory of meaning, and that a use theory of meaning does not need to take a form of a verificationist theory of meaning.

Therefore, it is worth to examine the strength of Dummett's fundamental assumption about the general form of a realist explanation of meaning. Our examination is a twofold task:

- Examination of the underlying reasons behind Dummett's assumption (sect. E4)
- Examination of Loar's meaning theoretic approach as a realist alternative to the realist truth-conditional theory of meaning (sect. E5)

4. Realism and Meaning

What is the reason behind Dummett's assumption that a realist conception of meaning has to be framed in terms of truth-conditions? As far as I am concerned, the relevant text where Dummett speaks about the relation between realism and a truth-conditional theory of meaning is the following one:

Realism consists in the belief that for any statement there must be something in virtue of which either it or its negation is true: it is only on the basis of this belief that we can justify the idea that truth and falsity play an essential rôle in the notion of the meaning of a statement, that the general form of an explanation of meaning is a statement of the truth-conditions.⁹⁴

The text says that a truth-conditional theory of meaning can be justified only on the basis of accepting realism. What particular belief constituting realism serves as the basis upon which a truth-conditional theory of meaning is justified? It is the belief "that for any statement there must be something in virtue of which either it or its negation is

⁹⁴ Dummett, "Truth", in: *Truth and Other Enigmas*, p. 14

true.” In this context, “any statement” is any statement belonging to the disputed class figuring in some R/AR dispute. Any such statement, according to Dummett’s realist, is either true or false “in virtue of a reality existing independently of us,” with further implications that any such statement possesses an objective truth-value “independently of our means of knowing it.” Moreover, the notion of the objective truth-value of any such statement should be understood in a very strong sense. As Bob Hale points out, in his interpretation of Dummett’s view:

The realist is to be understood as holding not merely that a statement may be true or false without our actually knowing its truth-value, nor even that a statement may be true or false even though we are in fact or in practice unable to tell which, but that there can be much more radical dislocation of truth-value and our capacity to recognition – a statement may possess a determinate truth-value without its being possible, even in principle, for us to come to know it.⁹⁵

According to Hale, it is precisely this belief in the statement’s possession of an objective truth-value, understood in a very strong sense as outlined above, that amounts to, or at least crucially involves, the truth-conditional thesis about meaning. Nevertheless, the pertinent question is: why does it amount to, or at least crucially involve, the truth-conditional thesis about meaning?

To adopt a realist view of any area of thought and talk is to conceive of its distinctive statements as endowed with meaning through being associated with evidentially unconstrained truth-conditions, that is, conditions whose satisfaction bears no essential connection, however attenuated, with the possibility of its being recognized by us.⁹⁶

A realist might agree with Hale’s above characterization of a realist view, nevertheless object to Hale’s reasoning that such characterization commits a realist to the truth-conditional thesis about meaning. Although a realist conceives statements of a given class as endowed with meaning in the way they are associated with evidentially unconstrained truth-conditions, this does not imply that their truth-conditions constitute their meaning. In a realistic interpretation of some statement, the meaning of the statement is that the *statement* is understood, among many things, as being associated with evidentially unconstrained truth-conditions. The association of *realist truth-conditions* with the statement is one of many aspects of the statement’s meaning, based

⁹⁵ Bob Hale, op. cit., p. 273

⁹⁶ Bob Hale, op. cit., p. 273

upon an interpretation of the world, e.g. “features of reality existing independent of us.” Furthermore, it might be that a general association of truth-conditions to a statement, an association including realist truth-conditions for some classes of statements, is constituted by already possessing an understanding of the statement, and not *vice versa*.

The point with the above objection is not in the strategy to provide what the alternative realist understanding of a statement consists in, but that it prompts for additional premises showing that Realism implies the Truth-Conditional Thesis.

Given a class of statements which are realistically interpreted, i.e. statements understood as possessing evidentially unconstrained truth-conditions, the cardinal question in the dispute is as follows: What is the grasp of the meaning of these statements consisting in unless the very association with their truth-conditions is the constitutive feature of the grasp of their meaning? This question is the main topic of the next section.

5. The Realist Reply

a) Rejection of the truth-conditional approach

We have noted the significance in Loar’s deflationist view about truth in his reply to ARC.

[D]eflationism about truth entails that meaning cannot be explained in terms of truth-conditions. (...) The Challenge of ARC is to explain what the understanding of the truth-conditions of statements consists in, when these truth-conditions of statements generally are not always capable of being recognized as obtaining. However, if a realist understanding is not a matter of grasping verification-transcendent truth-conditions, but in something else, the Challenge is met. “This something else” is knowledge of how to use statements. Loar’s article is devoted to show that a realist meaning theoretic framework can be faithful to the principles of a use theory of meaning, and that a use theory of meaning does not need to take a form of a verificationist theory of meaning.⁹⁷

The point about deflationism, namely that deflationism about truth entails that meaning cannot be explained in terms of truth-conditions, can be seen in Dummett’s considerations of a theory of meaning framed on the model of truth-definitions of Tarski’s kind, in his “What is a Theory of Meaning? (I).” The objective of such theory is to provide an explanation of what the meaning consists in by providing a representation of what it is that is known when an individual knows the meaning. On such a model, the

⁹⁷ Supra, p. 72-73

core of the theory of meaning are T-sentences derived from the theory of truth specifying the truth-conditions of the sentences belonging to the object language \mathcal{L} . The explanation of meaning of sentences belonging to \mathcal{L} takes a form of a direct ascription of meaning by M-sentences derivable from the theory. A M-sentence is derivable by converting a corresponding T-sentence “ \underline{s} is true iff S” into the form “ \underline{s} means S”, where $\underline{s} \in \mathcal{L}$ and S is a specification of the truth-conditions of \underline{s} . Dummett shows that such theory of meaning cannot succeed in its task to be a full-blooded theory of meaning. A full-blooded theory of meaning is a theory that is capable “to explain new concepts to someone who does not already have them,”⁹⁸ enabling us to understand the object language without presupposing a mastery of some another language. It is obvious that a M-sentence cannot explain the meaning of \underline{s} without presupposing the mastery of metalanguage. Consider the following M-sentence M_s .

M_s : “Sneg jeste beo” je istinit ako i samo ako sneg jeste beo.

The meaning of the sentence “sneg jeste beo”, belonging to the fragment language of Serbian, cannot be explained with M_s unless one has mastered the metalanguage to which M_s belongs, which in this case is Serbian not limited to the fragment language.

Likewise, it can be shown that using T-sentences would fail in the task to provide an explanation of meaning in terms of truth-conditions in a similar manner as M-sentences do. Consequently, the disquotational truth-schema is not sufficient in order to provide a full-blooded theoretic truth-conditional explanation of meaning. More generally, as we have seen in the exposition of the Manifestation Argument, understanding language cannot, in general, consist in explicit knowledge, i.e. knowledge manifested by *stating* the meaning of a sentence. T-sentences provide only explicit, verbalizable knowledge of truth-conditions.

Loar agrees with Dummett on the above considerations. Moreover, Loar contends that realism would be in deep trouble unless it could give a use theoretic account of

⁹⁸ Dummett, “What is a Theory of Meaning? (I)”, in: *The Seas of Language*, p. 5.

verification-transcendent truth-conditions. If a realist says that understanding of meaning consists in knowledge of truth-conditions, then

... that cannot mean “propositional” or, as Dummett puts it, “explicit” knowledge of truth-conditions, for that presupposes understanding the metalanguage in which that knowledge is couched, and what would we then say about that understanding? Dummett has the realist maintain that understanding consists in *implicit* knowledge of truth-conditions. But it is difficult to see what that could mean except some sort of use theory.⁹⁹

Loar, thus, agrees with Dummett that ARC cannot be answered by a truth-conditional meaning theoretic approach, but disagrees with Dummett’s contention that realism is committed to a truth-conditional theory of meaning. This disagreement is most interesting, since it prompts the question: “How is a use theoretic account of verification-transcendent truth-conditions possible such that it would meet the heart of the Challenge of ARC?”

Loar’s answer is that a holistic *conceptual role* theory can provide the desired use theoretic account. However, as we shall see, the holistic conceptual role theory is not sufficient unless it is coupled with our realist theory of nature. Suffice to say, the holistic character of the theory requires taking into account our background beliefs influencing our interpretation of the world. Neither is it sufficient to show how the realist conception of nature is a strongly persuasive worldview. It is required to provide a theoretic account of our concept-possession and among them the realist conception of the world. Therefore, Loar’s reply has two stages. The first one is a demonstration of how the realist conception of nature is a systematic and general view implied by our scientific theories. The second stage is an account of the conceptual role theory, which provides a theoretic account of our understanding concepts.

b) The first stage of argumentation

One important idea figuring in the first stage of argumentation is that realism is a modal claim.

Realism is a *modal* claim: it does not say that we are actually not in a position to verify such and such true statements, but that it could be the case that we were not in such a position, even given ideal inquiry. (...) The point of course is not *epistemological*: it is not that such and such statements are not conclusively verifiable and therefore like the statements Dummett regards as

⁹⁹ Loar, op. cit., p. 89.

subject to realist/anti-realist contention. There is a more fundamental ground for such a contention, concerning whether it *could* (logically could, not epistemologically could) happen that they were true, even though not, as in fact they are, verifiable by us.¹⁰⁰

Though we have verified that some sentence \underline{s} is true, it could happen that we would not recognize it as true due to various contingent circumstances.

The important contention of the argumentation is CNT.

[CNT]

It *could* (logically could, not epistemologically could) happen that statements were true, even though not, *as in fact they are*, verifiable by us.

Loar's point is that much of scientific research is subject to circumstances which are naturally contingent. Loar's strategy in showing that CNT is a valid premise consists in confronting an anti-realist with conceivable situations where verification of statements is subject to various contingencies. Statements that are in fact verified by us could happen not to be verified by us. To mention some of his examples: "our neural pathways are arranged in certain ways, the regions of space through which the relevant light or sound must travel lack distorting properties, and so on."¹⁰¹ An important conclusion, which Loar draws, is that "our general conception of the contingencies on which our knowledge depends implies that it is possible (...) that \underline{s} be true but not verifiable."¹⁰²

One important point, Loar being clear about, is that anti-realism cannot be refuted by finding some "counter-examples" to anti-realism. To mention one well-known example figuring in the R/AR dispute about the principle of bivalence, by considering a statement S_p , concerning some past state of affairs, and let us further assume that there in fact exists no evidence for or against S_p .

S_p : Nebuchadnezzar had a wart on his chin

For a realist, this example presents a powerful *intuitive* counter-example against semantic anti-realism, since for a realist it is impossible to deny the proposition P.

¹⁰⁰ Loar, op. cit., pp. 84-85.

¹⁰¹ Loar, op. cit., p. 97

¹⁰² Ibid

P: *Either* Nebuchadnezzar had a wart on his chin *or* he had not a wart on his chin

One of the past states of affairs about Nebuchadnezzar should have been obtained, although we do not know exactly which one. In our common linguistic practice, we have learned to regard any statement as true if the described state of affairs obtains, or regard the statement as not true unless the described state of affairs obtains. The same criterion for ascribing the truth-value to a sentence describing some present state of affairs also applies for sentences describing past states of affairs. Otherwise, many statements about the past for which even a semantic anti-realist would hold as true would lack a truth-value, e.g. statements about states of affairs that occurred yesterday and for which we have evidence. Consequently, S_p is *either* true *or* not true.

The above realist argument, although very intuitive, is not a strong one against semantic anti-realism. As Loar observes, “Dummett’s question about realism is not whether such a conviction is natural or even inevitable, *but rather whether the realist is entitled to it* [my emphasize].”¹⁰³ An intuitionist would correctly protest that a realist would not be entitled to reach the conclusion that S_p is either true or false.

Recall first what is the central issue in the semantic R/AR dispute. The issue is about the meaning of logical constants: how *should* we understand logical constants? In the mentioned realist’s objection above, the realist appealed to her *interpretation* of logical constants, which in this case are negation and disjunction. Moreover, the realist also appealed to her *interpretation* of how a truth-value of a statement is determined. However, the central dispute is about which *semantics of logic* is the appropriate one. The task for a semantic theory subservient to logic is to frame, “for each category of expression, a conception of the kind of *semantic value* that an expression of that category possesses.”¹⁰⁴ The semantic value of an expression is, to borrow Dummett’s words, “the feature of it that goes to determine the truth value of any sentence in which it occurs.”¹⁰⁵ For instance, a semantic value in Fregean semantics is a reference of a term. The

¹⁰³ Loar, op. cit, p. 98

¹⁰⁴ Dummett, *The Logical Basis of Metaphysics*, p. 24

¹⁰⁵ Ibid

semantic value of an expression is a more generalized concept where reference is one example.

So, *in the context of the dispute*, why is not a realist entitled to reach the conclusion that S_p is either true or not true? Because we are neither able to verify nor falsify the sentence, and thus, we cannot give the content to the supposition that it is either one or the other. It *might be* impossible to determine what truth-value it has; for instance, it might be that S_p has no truth-value at all. It might be that the singular term ‘Nebuchadnezzar’ lacks referent. Of course, the lack of referent is not the main reason for considering that S_p lacks a truth-value, since this is depending upon what kind of semantics we are adopting, whether we interpret our language as Russellian or not. However, the point lies precisely in the question of interpretation of logical constants and the kind of semantic value we assign to S_p .¹⁰⁶

A further point with considerations mentioned above concerning the question of the truth-value of S_p , is that our common sense intuitions, which are admittedly realist, are not sufficient in the resolution of the R/AR disputes. Given various conceptions reflected in our talk about the world, the important question is what the understanding of these concepts consists in. This question is not merely a question of clarification of the meanings of our concepts reflected in our talk. Rather, the question whether our talk about the world is grounded on legitimate application of concepts, as, for instance, the concept ‘truth’, is a substantial question for understanding the relation between language and world. By providing an account of what our concept-possession and, generally, our understanding of language consist in, we can, I believe, fruitfully approach the problems that are present in various R/AR disputes.

Given that we share strong realist intuitions, as reflected in Loar’s insights about the characterization of realism as a modal claim that were most eloquently argued for, we are interested in what does the realist conception of verification-transcendent truth-conditions consist in. Loar’s next stage of argumentation addresses this cardinal question of the dispute.

¹⁰⁶ For the intuitionist interpretation of logical constants, see *supra*, section A3, pp. 10-12

c) The second stage of argumentation

Loar regards his meaning theory as a use theory for the following reasons:

Such a theory is to be a “use” theory, that is, will explain understanding as the mastery of certain *ways of operating* verbally or conceptually, rather than in unexplicated terms of grasping truth-conditions or propositions.¹⁰⁷

Loar outlines two characterizations of the use theory. The positive one, characterizing it in the terms of what it does, namely that it explains understanding as the mastery of certain ways of operating verbally or conceptually. The negative one, characterizing it in the terms of what it does not, namely that it does not give an account of understanding in unexplicated terms. Dummett’s verificationist theory is a use theory, according to such characterization, since understanding of a sentence is explained in terms of mastering assertability conditions for the sentence. However, Loar characterizes his use theory as a *holistic conceptual role* theory, and sees it as an alternative to verificationism.

Generally, a conceptual role theory of meaning, as how Peacocke characterizes it, is a theory maintaining CP.

[CP]

For an expression to have a particular meaning is for it to have a certain role in its user’s psychology.¹⁰⁸

Conceptual role theories are often called internalist theories of content because meaning of an expression is regarded as being depended on factors internal to an agent. In Loar’s version, the kind of roles relevant in the determination of the meaning of an expression is in the form of dispositions.

A conceptual role theory holds that the meaning or content (...) of a sentence or thought consists in certain dispositions – its potential role in reasoning (in a network of such roles), in perception, in action guidance.¹⁰⁹

¹⁰⁷ Loar, op. cit., pp. 101-102

¹⁰⁸ Christopher Peacocke, “Holism”, in: *A Companion to the Philosophy of Language*, p. 229

¹⁰⁹ Loar, op. cit., p. 102

As such, it is a kind of reductionist theory of meaning or content: normative-semantic properties of expressions are explained in non-normative terms, e.g. dispositions, causal relations, etc.

Another, quite different, characterization of a conceptual role theory, encountered in the literature about cognitivism and philosophy of psychology, is that such a theory accepts CS.

[CS]

The meaning of an expression is determined by its role in our conceptual scheme.

A conceptual scheme is a (finite) set of fundamental concepts/beliefs/propositions, fundamental in the sense that they provide a framework for describing and explaining items of some subject matter. For our present purpose, we do not need to be concerned with what kind of concepts figures in the conceptual scheme, nor be concerned with the general question of what concepts are. The point is that, in this context, terms as ‘concepts’, ‘propositions’, etc., are used as neutrally as possible: whether they are some kind of mental images, some kind of Platonist entities, some special kind of dispositions, or another candidate crucial for the constitution of meaning/content of expressions/thoughts. However, the important point with concepts figuring in the conceptual scheme is that they, at least, involve an understanding of a certain set of rules of descriptions. These rules serve the purpose, among many things, to determine a structure of various conceptual relations, e.g. inferential relations. As a model of a conceptual scheme, Kant’s categories or Wittgensteinian philosophical grammar can serve as *examples*.

What is the relation between CP and CS? CP and CS are logically independent, i.e. neither of them entails each other.

CP does not imply CS, since an expression’s role in a speaker’s psychology could be, say, causally determined solely by internal factors not having any connection with factors determining other expressions. The idea of a conceptual scheme involves a structure of various conceptual *relations* between fundamental concepts/beliefs.

Neither does CS imply CP. A conceptual scheme does not necessarily involve any mental *process* or *state*, or, more generally, involve any (empirical) psychological

phenomenon. A conceptual scheme could be postulated as some kind of ideal Platonist Object, where meaning of some statement is a matter of Flash Grasping some Platonist Object.

No matter how implausible the idea of Flash Grasping or meaning-as-a-platonist-object may sound, the point is that as long as these ideas are present in the logical space of the possibilities involving CP and CS, CP and CS do not entail each other. We need additional premises that would exclude such conceived possibilities in order to establish some logical relation between CP and CS. However, we will assume that Loar would accept both CP and CS.

CS might be coupled with holistic ideas about language. However, whether a conceptual role theory is holistic, depends on the question of how much and in which manner the meaning of an expression is depending on other expressions in the language. If it is maintained that the meaning of an expression depends *constitutively* on its relation to *some* other expressions in the language, where these relations to other expressions are framed by our conceptual scheme, then such a theory is not global holistic. Global meaning holism, as Peacocke defines it, would be a position maintaining that the meaning of an expression depends *constitutively* on its relation to *all* other expressions in the language.

[GH]

The meaning of an expression depends constitutively on its relations to all other expressions in the language, where these relations may need to take accounts of such facts about the use of these other expressions as their relations to the non-linguistic world, to action and to perception.¹¹⁰

When speaking generally about holism, holism is, to borrow Prof. John Heil's words, "any view according to which properties of individual elements in a complex are taken to be determined by relations they bear to other elements. Holism is less a doctrine than a class of doctrines."¹¹¹ One can be a holist about meaning, without being a holist about justification. In order to distinguish holism about meaning from another type of holism, we will use the term 'meaning holism'.

¹¹⁰ Peacocke, op. cit., p. 227

¹¹¹ John Heil, a contributor to *The Oxford Companion to Philosophy*, "Holism", p. 371

Loar characterizes his conceptual role theory as holistic for the following reasons.

[My] theory of understanding (...) is 'holistic' because on it a statement's meaning is not some condition – e.g., a truth-condition or assertability condition – that can be specified independently of the statement's position within a network of statements.¹¹²

The crucial idea for meaning holism is that the meaning of a statement depends *constitutively* on its role and position within a network of other statements. Let us express this idea with H.

[H]

The meaning of any expression in the language depends constitutively by the expression's relation to other statements it has within a network of statements.

GH entails H, but not vice versa. H does not say anything about whether any expression is determined by the expression's relation to all or some statements. However, H is too weak in order to fully capture the idea of meaning holism, since H can render any theory of meaning as holistic. An understanding of any statement presupposes a possession of some concepts, for instance as the concepts figuring in a conceptual scheme, determining how the statement is related to other statements. Understanding the meaning of a term/statement does certainly depend on the understanding of *some* inferential relations the statement bears to *some* other terms/statements. For instance, understanding the term 'dog' requires the understanding of other terms like 'four', 'legs', 'animal', etc.

Thus, although meaning holism may not require an expression's meaning being determined by its relation to all other expressions, the expression should presumably relate to a significant range of expressions. How this significant range of expressions is specified depends on each meaning holist theory. It is not clear in Loar's account of his holistic conceptual role theory what range of expressions counts as significant.

Let it be noted that Loar, in his formulation, allows that the meaning of an expression, in some circumstances, can *also* be constituted by assertability conditions. What is not allowed is the idea that the meaning is solely constituted by assertability

¹¹² Loar, op. cit. p., 107.

conditions without considering an expression's place and the role it has in the network of other statements.

How does Loar's holistic conceptual role theory provide an answer to the Challenge of ARC?

Our mastery of that schematic theory of nature to which the argument [in the first stage of argumentation] appeals constitutes our conception of a verification-independent world. Understanding is in general constituted of a network of conceptual connections; those that make up our theory of nature imply that truth can transcend verifiability. (...) So there is a short answer to Dummett's request for a realist theory of understanding that satisfies the requirement of manifestability, namely: (i) our theory of nature implies the verificationist-independence of facts about the natural world, (ii) the holistic conceptual role theory of understanding makes our mastery of that theory sufficient for our having a bona fide conception of verification-independent states of affairs, and (iii) that realist conception of ours is manifestable in behavior for the simple reason that the relevant conceptual role are manifestable in our *verbal* behavior.¹¹³

We see how both stages of Loar's argumentation are connected. The objective of the first stage of argumentation has been to demonstrate that our general conception of nature is realist. However, the first stage is not sufficient for showing that realism is a tenable position unless it can account for what the understanding of the verification-transcendent truth-conditions consists in; an account constrained within a use theoretic framework. This is the task of the second stage of argumentation.

However, Loar's conclusion of the first stage of argumentation plays an important role in the second stage of the argumentation. Given that we have a realist holist conceptual scheme governing our theory of nature, the meaning of a statement consists in the role it has in the conceptual scheme. Thus, the meaning of a statement about verification-transcendent states of affairs, likewise, consists in having such a role in a *realist* conceptual scheme. Furthermore, the role it has in the conceptual scheme, i.e. the relation it bears with other *expressions*, are manifestable in our *verbal* behavior.

The above argument, as Loar remarks, is just a short answer to the Challenge of ARC, serving as a preliminary basis of the argumentation. Therefore, Loar further develops a more detailed outline of the main ideas introduced in the short answer, addressing various anticipated objections, which is the topic of the next section.

¹¹³ Loar, op. cit., pp. 102-103

6. Evaluation of Loar's Reply

Let us first examine an objection concerning the relation between conceptual roles and assertability.

The conceptual role of a statement is a matter of its inferential properties and responsiveness to perception. That could seem to imply that the meaning of a statement consists in a certain set of *assertability conditions*. But how then could \underline{s} 's truth be understood as independent of \underline{s} 's assertability, given that assertability conditions constitutes \underline{s} 's meaning?¹¹⁴

Loar's formulation of the problem is right to the point, expressing the core of Dummett's Challenge. It is required of Loar to spell out more details about his holistic conceptual role theory such that it would elucidate the relation between the conceptual role of a statement and the statement's assertability conditions in favor for realism.

Loar's longer answer to the Challenge of ARC provides a more thorough account of his holistic theory. Statements are divided into two classes with respect to their assertability conditions:

- a) Theoretical statements: Statements whose assertability conditions are exclusively holistic.
- b) Observational statements: Statements whose assertability conditions can *also* have non-holistic, perceptual, assertability conditions.

Assertability conditions of a statement \underline{s} are "the enormous array of all combinations of observational and theoretical statements that would support \underline{s} ."¹¹⁵ Holistic assertability conditions of a statement \underline{s} are conditions constituted only by the inferential relations \underline{s} bears to other statements such that they would support \underline{s} . Although theoretical statements are inferentially connected to observational statements, they are not thereby endowed with perceptual assertability conditions. What is the relation between the conceptual role of a theoretical statement \underline{s} and \underline{s} 's holistic assertability condition? Loar is of the opinion that the \underline{s} 's conceptual role determines \underline{s} 's holistic assertability conditions, and not *vice versa*. However, as Loar points out, whether this is correct depends on the acceptance of an *a priori* holistic confirmation theory, i.e. a theory that would assign to a conceptual role such an array of assertability conditions independently

¹¹⁴ Loar, op. cit., p. 103

¹¹⁵ Loar, op. cit., p. 104

of empirical considerations on our belief-forming procedures. If such an *a priori* holistic theory were not *generally* tenable for the whole language, what would the relation between the conceptual role of a theoretical statement \underline{s} and \underline{s} 's assertability conditions be? According to Loar, "a conceptual role neither determines nor is determined by a set of assertability condition."¹¹⁶ Loar offers the following argument (Arg6) for his contention.

[Arg6]

Indeed, we can turn any such anti-realist argument that presupposes an *a priori* confirmation theory on its head. If there were such a theory, it would imply that meaning (conceptual role) is exhausted by assertability conditions. But that cannot be so, given the coherence of the conceptual role of ' \underline{s} is true but unassertable'. Therefore, no such a confirmation theory is correct.¹¹⁷

In Arg6, Loar is most concerned with the question whether an *a priori* confirmation theory could support anti-realism, i.e. supporting the view that the conceptual role of theoretical statement \underline{s} is exhaustively determined by \underline{s} 's assertability conditions. The target of the argument is an anti-realist *a priori* confirmation theory. Let us say that it wouldn't. Why is this relevant considering if we reject an *a priori* confirmation theory? (The argument appears in the footnote commenting the sentence "But, if not, a conceptual role neither determines nor is determined by a set of assertability condition"). The argument is relevant because we are presently considering only the class of *theoretical* statements of the language. Loar's contention is actually that the conceptual role of a theoretical statement \underline{s} determines \underline{s} 's assertability. Therefore, the question arises, when considering only the class of theoretical statements, whether an *a priori* realist confirmation theory could be tenable.

The class of theoretical statements could be viewed, roughly, as a fragment of the language for which an *a priori* confirmation theory would be most suitable. The only threat against the realist contention, in this context, concerning the class of theoretical statements is an anti-realist *a priori* confirmation theory. Since, according to Arg6, an *a priori* confirmation theory does not support anti-realism, a rejection of such theory, conceived as the only threat, would not have any bearing to the question whether the role

¹¹⁶ Loar, op. cit., p. 104

¹¹⁷ Loar, op. cit., footnote 16, p. 115

of a theoretical statement is determined by \underline{s} 's assertability conditions. Loar, therefore, accepts either A or B, concerning the relation \mathcal{R}_τ of the conceptual role ρ of a theoretical statement \underline{s} and \underline{s} 's assertability conditions α .

A: ρ determines α

B: neither ρ determines α , nor α determines ρ

However B is problematic, since it is quite unclear how α is determined at all, if both A and the verificationist view on \mathcal{R}_τ are denied. If Loar's holistic realist theory would have some plausibility, it is better to hold A. For these reasons, we assume that Loar favors A.

The structure of Arg6 is of a *reductio ad absurdum* kind of the form Φ .

$\Phi: [(P \rightarrow \perp) \therefore \neg P]$

Although it is susceptible to use *reductio ad absurdum* arguments in the context of the dispute with an intuitionist, Φ is a valid schema in the intuitionistic logic.¹¹⁸ The problem is not in the structure of Loar's argument, but in the premises. Let us take a closer look on the argument.

[ArgL]

$P_1: T_{ar}$

$P_2: T_{ar} \rightarrow M_{ac}$

$P_3: M_{ac} \rightarrow \neg C_{cr}$

$P_4: C_{cr}$

$P_5: C_{cr} \wedge \neg C_{cr}$

T_{ar} : Anti-realist confirmatory theory

M_{ac} : Meaning is exhausted by Assert. Cond.

C_{cr} : Coherence of realist meta-statements

$\therefore \neg T_{ar}$

¹¹⁸ However, the schema $[(\neg P \rightarrow \perp) \therefore P]$ would not be valid in the intuitionistic logic. A *reductio ad absurdum* argument is based on the law of double negation, i.e. $[\neg\neg P \leftrightarrow P]$. However, in the intuitionistic logic, only $[P \rightarrow \neg\neg P]$ can be unrestrictedly used. The law of double negation for a statement P is allowed if the law of excluded middle is established for P , i.e. $[P \vee \neg P \therefore \neg\neg P \rightarrow P]$ is valid in the intuitionistic logic. The principle of bivalence, which is the very principle under the question, is the semantic principle to the corresponding principle of the law of excluded middle. The denial of the law of excluded middle leads to the denial of the principle of bivalence, (but not converse). See more about the intuitionistic meaning of logical constants in: Dummett, *Elements of intuitionism*, pp. 9-31.

P_1 , P_2 and P_4 are not disputable. What is disputable is the premise P_3 , which coupled with undisputable premises yields a contradiction expressed by P_5 . An anti-realist confirmatory theory does not necessarily dispute the *coherence* of realist *meta*-statements, e.g. ‘ \underline{s} is true but unassertable’, of a realist theory of language. Rather, an anti-realist disputes the realist interpretation of \underline{s} . Therefore, the premise P_3 is illegitimate to use in the context of the dispute.

Loar’s continuation of the account of his holistic conceptual role theory addresses the question of the relation \mathcal{R}_ω between the meaning (conceptual role) and the assertability conditions of an observational statement. What is, then, the relation \mathcal{R}_ω ? Let us consider the statement “there is a cat on the mat.”

Is my conception of a cat then, simply whatever it is that triggers my recognitional capacities of the \underline{x} that underlies my cat perceptions? Evidently not, for the conceptual roles of my thoughts about cats are rich and complex, fitting them into a spatiotemporal framework, giving them a variety of causal powers, of compositional structures, and so on. The virtue of the conceptual role theory of meaning is that it makes sense of this traditional realist idea (...). The fundamental point is this: I can conceive of \underline{s} ’s being true independently of my being able to recognize it as true, even though that recognitional ability is part of \underline{s} ’s conceptual role, because I have mastered a theory of nature in which that recognitional ability is only contingently connected with the truth of \underline{s} .¹¹⁹

We observe a circularity in the argumentation: [‘theory of nature’ \rightarrow ‘holistic conceptual role theory’ \rightarrow ‘theory of nature’]. The mastery of the realist theory of nature is accountable with conceptual roles, which in their turn are based upon this mastery of the realist theory of nature. However, is this circularity of a vicious kind? Yes, it is unless there is some further aspects of either ‘the conceptual role of \underline{s} ’ justifying an appeal to ‘the theory of nature’, or *vice versa*.

A serious problem threatening the first disjunct, i.e. ‘the conceptual role of \underline{s} ’, is that a holist network of statements in the language is immune to, what Dummett calls, revisionism. This problem of revisionism is connected to objections targeted generally against use theories.

¹¹⁹ Loar, op. cit., p. 105

It might seem that an approach to meaning which regard it as exhaustively determined by use would rule out any form of revisionism. If use constitutes meaning, then, it might seem, use is beyond criticism.¹²⁰

This problem is acute when considering the case with the “pure mathematical holism.” In such a case, there can be no place for rejecting any established mathematical practice. For instance, the use of certain arguments cannot be subject to criticism, since the use of arguments, *per ex hypothesis*, is constitutive of the meanings of mathematical statements. According to Dummett, the problem of the immunity against revisionism is present only in holistic use theories.

Such an attitude [against any form of revisionism] is one possible development of the thesis that use exhaustively determines meaning: it is, however, one which can, ultimately, be supported only by the adoption of a holistic view of language. On such a view, it is illegitimate to ask after the content of any single statement, or even after that of any theory, say a mathematical or a physical theory.¹²¹

As we have seen in Loar’s holism, the conceptual role of a statement, especially a statement belonging to the theoretical class, determines assertability conditions of the statement. Loar’s account of observational statements appealed to the holist conceptual roles of the statements, where the non-holist aspect of assertability conditions was minimized in the relation to the holistic assertability conditions. Such a tendency can easily be developed into conservatism. Loar is clear about the problem when discussing the case of holism in the philosophy of science, where the term’s meaning, for a person \underline{x} , depends on its place in the overall theory that \underline{x} actually accepts.

But ‘holism’ would, I suppose, be allowed to cover also the potential responsiveness of a sentence to certain new evidence, how its acceptance would affect other beliefs, and so on; for otherwise holism would have nothing to say about the meaning of sentences that are frameable in \underline{x} ’s language but not actually accepted by \underline{x} .¹²²

Loar’s point is that the holistic assertability conditions permit the change of beliefs based on *new* evidence. However, if so, the relation between conceptual roles and assertability conditions is not so asymmetric as it usually is in various holistic theories. Loar’s holist theory, in that regard, resembles more Dummett’s molecular view of

¹²⁰ Dummett, “The Philosophical Basis of Intuitionistic Logic”, p. 218

¹²¹ Dummett, op. cit., p. 218

¹²² Loar, op. cit., p. 102

language. However, this topic is quite complicated, and it cannot be covered in the scope of the thesis. One important question is how much minimized is the role of non-holistic assertability conditions in the relation to holistic assertability conditions in order to escape the charge of conservatism.

Since much of Loar's theory of understanding hinges upon the argumentation of the first stage, let us consider again the second disjunct. As we have seen, an important point with the argument is that realism is a modal claim, which is most forcefully formulated in the contention CNT. Loar's examples about statements involving states of affairs in inaccessible regions of space, e.g. the statement about Sirius, illustrate both the realist modal claim and the plausibility of the view that there is a gap between 'being true' and 'being known to be true'.

However, in spite of the realist plausible view on the contingencies affecting our verifications, is a realist entitled to postulate the truth of any statement without thereby making any assumptions about the statement's assertability conditions? Dummett's answer is negative due to one problem not yet resolved in the dispute. Although it is quite easy for a realist to describe the state of affairs illustrating the gap between 'being true' and 'being known to be true', nevertheless, both parties in the dispute have to solve one problem.

It is very easy to describe a hypothetical state of affairs, in which a certain statement held good, and a subsequent chain of events, consonant with existing scientific theory, which irrevocably deprived us of any means of determining that that statement was true. That is not the problem. The problem is how far we are entitled to postulate that a state of affairs obtain of which we have no inkling and shall never have any. All varieties of anti-realism draw bounds to such postulation.¹²³

Both realists and anti-realists share this problem. The problem we see here is similar when considering the dispute between a Platonist and an Intuitionist: we are again involved with the problem of "judgments of possibilities." In both disputes, realists and anti-realists appeal to their respective modal notions.

Can, then, some resolution to the dispute be found? Dummett is optimistic on this score. His proposal is interesting in that it acknowledges a gap between 'being true' and 'being known to be true'.

¹²³ Dummett, "Reply to Brian Loar", in: *Michael Dummett: Contributions to Philosophy*, p.276

[A]ny plausible theory must allow for a fairly wide gap between being true and being known to be true, global anti-realism needs, if it is to avoid implausibility, to find some other account of how the gap occurs, and if it not to surrender to realism, to place a bound on its width. It is an enquiry into this question that the resolution of the dispute between realist and anti-realist is likely to be found.¹²⁴

His proposal for the *resolution* of the dispute might sound strange, in that the answer to the question of how the gap occurs seems to be found within the realist framework, and that the burden of the enquiry in the proposed question lies heavily on the anti-realist. However, recall that the realist is also faced with the problem of “how far we are entitled to postulate that a state of affairs obtain of which we have no inkling and shall never have.” In other words, a realist is also faced with the question of the gap between ‘being true’ and ‘being known to be true’. A question that is not yet resolved in the dispute.

¹²⁴ Dummett, “Reply to Loar”, p. 276

CONCLUSION

In our discussion about the Semantic Anti-Realist Challenge, we have been concerned with two main questions:

- The validity of the underlying principles of ARC
- The force of ARC

The first question is concerned with the cogency of the arguments for the underlying principles of ARC. The second question is concerned with whether the Challenge of ARC can be met by a realist given that the principles of ARC are accepted.

In our exposition and evaluation of Dummett's three arguments for the underlying principle of ARC, we have observed that the Communication argument plays a significant role in the relation to other two arguments, the Manifestation and the Acquisition argument, in that it strengthens the two other arguments. The weakness of the Manifestation argument is that it does not point out how the implicit knowledge is manifested in such a way that it is shown that the manifestation is a constitutive feature of the speaker's grasp of statements. However, the feature of communication accounted by the Communication Argument might further elucidate the idea of the Manifestation Argument, namely that the manifestation of the speaker's knowledge is a constitutive feature of the speaker's grasp of statements. Likewise, the Communication Argument can be a complimentary account to the Acquisition Argument, if the Acquisition argument does not provide an exhaustive account of what the understanding of language consists in. The Communication Argument is seen as a strong argument when dealing with a public language. In the context of the dispute between a Platonist and Intuitionist, the Communication Argument is, thus seen as, highly relevant in that the language of mathematics is a public language.

The fundamental underlying principle of ARC is the use principle (UP). Any theory of meaning aspiring to have a character of a use theory has to account for the meaning of statements possessing verification-transcendent truth-conditions. ARC is a challenge prompting for an alternative non-verificationist account of verification-transcendent truth-conditions. We have been concerned with two kinds of disputes where

UP was the acknowledged principle: (1) a dispute between a Platonist (of the second type) and an Intuitionist, and (2) a dispute between a Realist endorsing a Holistic Realist theory of meaning, and a semantic anti-realist. In the context of the first dispute, the Challenge is not met. However, one recurring theme, introduced in the first dispute, emerging in the second one, is the theme of “judgments of possibility.” Realism was characterized as a modal claim: It *could* happen that statements were true, even though not, *as in fact they are*, verifiable by us. This recurring theme of the judgments of possibility makes the dispute very difficult to resolve. Judgments of possibility are much grounded on our intuitions. However, our intuitions about what is possible/impossible cannot provide a sufficient guidance, in that our various intuitions are notoriously different. Much of the second realist reply is grounded upon realist intuitions about possible state of affairs. This, however, does not invalidate the second realist reply. It prompts for further philosophical investigation on the questions of the gap between ‘being true’ and ‘being known to be true’. Questions that are not yet resolved in the dispute.

BIBLIOGRAPHY

Works:

Dummett, Michael: *The Logical Basis of Metaphysic*. Gerald Duckworth & Company Limited, London, 1995.

Dummett, Michael: *The Elements of Intuitionism*. Clarendon Press, Oxford, 1977.

Articles:

Craig, Edward: "Meaning, Use and Privacy" in: *Mind* (1982), Vol. 91.

Dummett, Michael: "The Philosophical Basis of Intuitionistic Logic" in: *The Truth and Other Enigmas*. Gerald Duckworth & Company Limited, London, 1978.

Dummett, Michael: "Reply to Brian Loar" in: *Michael Dummett: Contributions to Philosophy*. Ed. Barry Taylor; Martinus Nijhoff Publishers, Dordrecht, 1987.

Dummett, Michael: "What is a Theory of Meaning? (I)", in: *The Seas of Language*, Clarendon Press, Oxford, 1997.

Dummett, Michael: Preface to *The Truth and Other Enigmas*. Gerald Duckworth & Company Limited, London, 1978.

George, Alexander: "How not to refute realism" in: *The Journal of Philosophy*, vol. XC, No. 2, Feb. 1993.

Hale, Bob: "Realism and its oppositions" in *A Companion to the Philosophy of Language*. Ed. Bob Hale and Crispin Wright; Blackwell Publishers, 1999.

Loar, Brian: "Truth Beyond All Verification" in: *Michael Dummett: Contributions to Philosophy*. Ed. Barry Taylor; Martinus Nijhoff Publishers, Dordrecht, 1987.

Peacocke, Christopher: "Holism" in *A Companion to the Philosophy of Language*. Ed. Bob Hale and Crispin Wright; Blackwell Publishers, 1999.

Skorupski, John: "Meaning, use, verification" in *A Companion to the Philosophy of Language*. Ed. Bob Hale and Crispin Wright; Blackwell Publishers, 1999.